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Electric and Magnetic Forces: A Direct Calculation. II*

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In Part 3, the principles developed in the preceding parts are illustrated by applying them to the problem of a sphere, with arbitrary material properties, immersed in a fluid with linear dielectric properties, in an applied electric field that is symmetric about an axis but is otherwise arbitrary. The long-range electric force and the short-range force due to fluid pressure are calculated separately; the sum of these is what is usually calculated by the maxwell stress method, and the resolution into two terms is admittedly not unique. In Part 4, some general work and energy relations are developed; these are put into a form such that elementary thermodynamic principles can be applied directly, without resort to physical interpretations of the macroscopic field vectors. The application to magnetoelastic and piezoelectric phenomena is indicated. The standard formulas of piezoelectricity, as developed by Voigt, are shown to be correct only if the "stresses" occurring in them are given a particular one of several possible interpretations.

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LIST OF PRINCIPAL SYMBOLS

$_A$ = (subscript) of a Class A fluid.
$\mathbf{B}$ = magnetic flux density.
c = speed of electromagnetic waves.
$\mathbf{D}$ = electric flux density; here defined (for all units) as $\mathbf{E}+\gamma\mathbf{P}$.
D/dt = rate of change in axes moving and rotating with mass element.
d/dt = rate of change in axes moving but not rotating with mass element ($= (D/dt) + \boldsymbol{\Omega}\times$, for a vector; $= (\partial/\partial t) + \mathbf{v}\cdot\nabla$, for a vector or scalar).
$\partial/\partial t$ = rate of change in fixed axes.
$\mathbf{E}$ = electric intensity.
de_{xx}, de_{xy}, etc. = differential elements of strain: $de_{xx} = \epsilon_{xx}dt$.
e_{xx}, e_{xy}, etc., = small strains (in elastic solids).

* Part I (Parts 1 and 2) appeared in Am. J. Phys. 19, 290–304 (1951). In the abstract, for I and II read Part 1 and Part 2.

F, F', F, F' = thermodynamic potentials with polarizations and strains as independent variables.

$\mathbf{F}$ = force; $\mathbf{F}_e(\mathbf{F}_m)$ electric (magnetic) force.

$\mathbf{F}_\nu$ = traction across surface with normal $\mathbf{n}$.

$\mathbf{f}$ = acceleration; f_x, f_y, f_z components of acceleration.

$\mathbf{H}$ = magnetic intensity; here defined (for all units) as $\mathbf{B}-\gamma\mathbf{M}$.

$\mathbf{J}_c, \mathbf{J}_m, \mathbf{J}_t$ = volume density of conduction current, amperian current, and total current, respectively.

$\mathbf{L}$ = torque.

l, m, n = direction cosines of normal $\mathbf{n}$.

$\mathbf{m}$ = magnetic moment.

$\mathbf{M}$ = magnetic polarization (magnetic moment per unit volume; magnetization).

$\mathbf{M}$ = specific magnetic polarization (magnetic moment per unit mass).

m = mass; dm, element of mass.

$\mathbf{n}$ = outward normal to surface.

$_n$ = (subscript) normal component.

$\mathbf{p}$ = electric moment.

$\mathbf{P}$ = electric polarization (electric moment per unit volume).

$\mathbf{P}$ = specific electric polarization (electric moment per unit mass).

p_ν = pressure across surface with normal $\mathbf{n}$; p', p_0, $p(\rho)$, pressures of various kinds.

dp = element of magnetic pole strength.

$\mathbf{r}$ = position vector or mutual distance vector ($=r\mathbf{1}$).

r = magnitude of $\mathbf{r}$.

S = surface; dS, element of surface area.

T = kinetic energy.

t = time.

$_t$ = (subscript) tangential component.

U_m, U_m' = "magnetic" energies.

V = potential of the mechanical body force per unit mass, or electrostatic potential (as specified).

V = specific volume ($=1/\rho$).

$\mathbf{v}$ = velocity of a mass element.

v_x, v_y, v_z = components of $\mathbf{v}$.

dW = differential element of work.

X, Y, Z = components of mechanical body force per unit mass.

X_x, X_y, etc., = stress components: X_y is the x-component of stress across a surface with normal along y.

X_ν, Y_ν, Z_ν = components of traction $\mathbf{F}_\nu$ across a surface with normal $\mathbf{n}$.

Z, Z', Z, Z' = thermodynamic potentials with field vectors and strains as independent variables.

α = a constant between 0 and 1, otherwise arbitrary.

γ = a constant dependent on the units: $=4\pi$ for gaussian, $=1$ for Lorentz-Heaviside, $=4\pi\times10^{-7}c^2$ for mks-coulomb.

$\epsilon_{xx}, \epsilon_{xy}$, etc. = velocity strains: $\epsilon_{xx}=\partial v_x/\partial x$; $\epsilon_{xy}=(\partial v_y/\partial x)+\partial v_x/\partial y$.

κ_e = (relative) dielectric constant ($=D/E=1+\gamma\chi_e$).

κ_m = (relative) magnetic permeability ($=B/H=1+\gamma\chi_m$).

$_\nu$ = (subscript) across a surface with normal $\mathbf{n}$.

ρ = mass density.

ρ_m = volume density of magnetic poles.

ρ_c, ρ_p, ρ_t = volume density of conduction charge, polarization charge, and total charge, respectively.

τ = volume; $d\tau$, element of volume.

χ_e = electric susceptibility ($=P/E$).

χ_m = magnetic susceptibility ($=M/H$).

$\mathbf{\Omega}$ = angular velocity of a mass element ($=\frac{1}{2}\nabla\times\mathbf{v}$).

$d\omega$ = differential element of rotation ($=\mathbf{\Omega}dt$).

ω = small rotation (in elastic solids).

$\mathbf{1}$ = unit vector in the direction of $\mathbf{r}$.

THE relation of Parts 3 and 4 to the preceding Parts has been indicated in Sec. 1.2. In Part 3, no new principles are introduced, but those already developed are illustrated by detailed analysis of a particular problem. In Part 4, the principles of Part 1, which are independent of energy considerations, are supplemented by energy arguments (of a more restricted type than those usually encountered), in order to obtain formulas applicable to a solid material. The "Report" occasionally cited is, as before, that

of the Coulomb's Law Committee of the AAPT (see reference 1, Part I). The reader is reminded that the equations of this article reduce to those of the gaussian unit system if the constant γ is replaced by 4π and to those of the Lorentz-Heaviside system if it is replaced by 1; and that the substitution $\gamma/4\pi c^2 = 10^{-7}$ newton/amp^2 leads to a set of equations consistent with use of mks mechanical units and of the volt and ampere, but not consistent with the currently prevalent mks-Giorgi definitions of **D**, **B**, **H**, and **M** (see Sec. 1.2).

3. FORCES ON A CHARGED SPHERE

3.1 Statement of the Problem

The problem to be considered here is the following. A sphere of radius a, with center at $x=y=z=0$, contains electric charges distributed symmetrically about the z axis but arbitrarily in a plane through this axis. These charges may include a central point charge, ring charges, volume distributions, and distributions on the surface of the sphere and on surfaces inside the sphere. They may include conduction charges and polarization charges. A surrounding Class A fluid is subjected to an external electric field intensity $\mathbf{E}'$ (defined as in the Report, p. 10), symmetrical about the z axis but otherwise arbitrary. The dielectric constant of the fluid, $(\kappa_e)_A$, will be represented for simplicity by K. It is required to find the long-range electric force on the sphere, the short-range force due to fluid pressure, and the resultant force.

This includes, as special cases, a conducting sphere and a dielectric sphere; these special cases will be considered later. For the present, the charge distribution in the sphere is supposed to be given; if any polarization is present, it is supposed to be taken account of by explicitly including, in the charges, the volume and surface densities $-\nabla\cdot\mathbf{P}$ and $\mathbf{n}\cdot\mathbf{P}$ contributed by the polarization. This is permissible in the calculation of the long-range force; and short-range interactions between parts of the sphere need not enter in this problem.

3.2 Solution of the Potential Problem

The first step is to evaluate the electrostatic potential V, from which the electric intensity can be calculated. Point, ring, and surface charges in and on the sphere may be regarded as limiting cases of a continuous volume density ρ_t; with this simplification, V satisfies the equation $\nabla^2 V = -\gamma\rho_t$ for $r<a$ and Laplace's equation for $r>a$, reduces to the potential of the external field at large r, is continuous at $r=a$, and has there a discontinuity in its radial derivative such that the internal value is K times the external value.

The potential of the external field must be of the form

$$V' = \sum_{n=0}^{\infty} V_n r^n P_n(\cos\theta), \qquad (3.2\text{–}1)$$

where (r, θ, ϕ) are spherical coordinates with Oz as polar axis, and the V_n's are given constants. In the presence of the sphere, the potential for $r>a$ must be of the form

$$V_{(2)} = \sum_{n=0}^{\infty} (V_n r^n + B_n/r^{n+1}) P_n(\cos\theta). \qquad (3.2\text{–}2)$$

The potential of the charge distribution ρ_t alone may be found by integrating the potential of a charged ring.[13] This gives, for $r<a$,

$$V_{Q(1)} = (\gamma/4\pi) \sum_{n=0}^{\infty} f_n(r) P_n(\cos\theta), \qquad (3.2\text{–}3)$$

where

$$f_n(r) = 2\pi \int_0^{\pi} P_n(\cos\theta')\sin\theta' d\theta' \times \left[\frac{1}{r^{n+1}}\int_0^r \rho_t(r', \theta') r'^{n+2} dr' + r^n \int_r^a \rho_t(r', \theta') \frac{1}{r'^{n-1}} dr'\right]; \qquad (3.2\text{–}4)$$

$\rho_t(r', \theta')$ is the total charge density at (r', θ', ϕ'). Point, ring, and surface charges may be included as special cases of ρ_t. For $r>a$, the second integral in $f_n(r)$ is absent and the first has limits 0 and a, so that

$$V_{Q(2)} = \frac{\gamma}{4\pi} \sum_{n=0}^{\infty} \frac{M_n}{r^{n+1}} P_n(\cos\theta), \qquad (3.2\text{–}5)$$

[13] W. R. Smythe, *Static and Dynamic Electricity* (McGraw-Hill Book Company, Inc., New York, 1939), pp. 137–138.

where

$$M_n = 2\pi \int_0^\pi P_n(\cos\theta')\sin\theta' d\theta' \int_0^a \rho_t(r', \theta') r'^{n+2} dr'$$

$$= \int \rho_t(r', \theta') r'^n P_n(\cos\theta') d\tau'; \qquad (3.2\text{–}6)$$

the volume integral is extended over the interior of the sphere. The quantities M_n are moments of charge; in particular,

$$M_0 = \int \rho_t(r', \theta') d\tau' = \text{total charge} = q, \qquad (3.2\text{–}7)$$

and

$$M_1 = \int \rho_t(r', \theta') z' d\tau' = z\text{-component of electric moment} = p_z. \qquad (3.2\text{–}8)$$

The potential of the whole system (sphere, fluid, and external sources) must, for $r<a$, be of the form

$$V_{(1)} = V_{Q(1)} + \sum_{n=0}^{\infty} A_n r^n P_n(\cos\theta). \qquad (3.2\text{–}9)$$

The constants A_n and B_n are determined by the boundary conditions at $r=a$. The result of evaluating the constants is

$$V_{(1)} = V_{Q(1)} + \sum_{n=0}^{\infty} \frac{1}{(n+1)K+n}\left[(2n+1)KV_n - \frac{\gamma}{4\pi}(n+1)(K-1)\frac{M_n}{a^{2n+1}}\right] r^n P_n(\cos\theta), \qquad (3.2\text{–}10)$$

$$V_{(2)} = \sum_{n=0}^{\infty}\left\{ V_n r^n + \frac{1}{(n+1)K+n}\left[n(K-1)a^{2n+1}V_n + \frac{\gamma}{4\pi}(2n+1)M_n\right]\frac{1}{r^{n+1}}\right\} P_n(\cos\theta). \qquad (3.2\text{–}11)$$

3.3 The Long-Range Force

The second step is the evaluation of the long-range electric force. This requires knowledge of the z-component of the electric field intensity produced in the sphere by sources outside the sphere; this field intensity may be evaluated by subtracting from the whole electric intensity, $-\partial V/\partial z$, the electric intensity of the internal charges, $-\partial V_{Q(1)}/\partial z$. This gives

$$E_{0z} = -\sum_{n=1}^{\infty} \frac{n}{(n+1)K+n}\left[(2n+1)KV_n - \frac{\gamma}{4\pi}(n+1)(K-1)\frac{M_n}{a^{2n+1}}\right] \cdot r^{n-1}P_{n-1}(\cos\theta), \qquad (3.3\text{–}1)$$

since

$$(\partial/\partial z)[r^n P_n(\cos\theta)] = n r^{n-1} P_{n-1}(\cos\theta) \qquad (3.3\text{–}2)$$

and the term $n=0$ vanishes. The electric force on the sphere (which by symmetry is along z) is found by multiplying E_{0z} by $\rho_t(r, \theta)$ and integrating over the interior of the sphere. But

$$\int \rho_t(r, \theta) r^{n-1} P_{n-1}(\cos\theta) d\tau = M_{n-1}, \qquad (3.3\text{–}3)$$

and therefore the long-range force is

$$F_1 = -\sum_{n=1}^{\infty} \frac{n}{(n+1)K+n}\left[(2n+1)KV_n - \frac{\gamma}{4\pi}(n+1)(K-1)\frac{M_n}{a^{2n+1}}\right] M_{n-1}$$

$$= \sum_{n=0}^{\infty}\left[-\frac{(n+1)(2n+3)KV_{n+1}M_n}{(n+2)K+(n+1)} + \frac{\gamma}{4\pi}\frac{(n+1)(n+2)(K-1)M_{n+1}M_n}{[(n+2)K+(n+1)]a^{2n+3}}\right]. \qquad (3.3\text{–}4)$$

The terms in Eq. (3.3–4) are of two types. The terms containing products $V_{n+1}M_n$ are forces exerted on the charges by the external sources and by the part of the fluid polarization induced by them; the leading term is

$$-\frac{3K}{2K+1}V_1 M_0 = +\frac{3K}{2K+1}E_z'^0 q, \qquad (3.3\text{–}5)$$

where $E_z'^0$ is the z-component of external electric intensity at (0, 0, 0). Since it is known (from the energy method or from the calculations of Part 2) that the total force in a uniform field is $E_z'^0 q$, there must be a term in the short-range force which, when added to the term just derived, will give $E_z'^0 q$. The terms containing products $M_{n+1}M_n$ are independent of the external field.

The leading term is the one in $M_1M_0=p_sq$. Its presence means that the dipole moment of the sphere induces polarization in the fluid, that this polarization produces a uniform field in the sphere, and that therefore the polarization induced by the dipole moment exerts a force on the sphere if the sphere has a net charge. Since the energy of the charged sphere and fluid is (in the absence of an external field) independent of the position of the sphere in the fluid, the resultant force in no external field must vanish; there must therefore be terms in the short-range force that will cancel all the terms of this type in the long-range force.

3.4 The Short-Range Force

The third step, preliminary to the evaluation of the short-range force, is the evaluation of the fluid pressure p_r at the surface of the sphere. It is given by Eq. (2.3–13), to which Eq. (2.3–12) reduces for a Class A fluid (see Sec. 2.4):

$$p_r=p_0+\frac{K-1}{2\gamma}(KE_r^2+E_\theta^2). \tag{3.4–1}$$

Differentiation of Eq. (3.2–11) gives at $r=a+0$

$$E_r=\sum_{n=0}^{\infty} C_nP_n(\cos\theta), \tag{3.4–2}$$

$$E_\theta=\sum_{n=0}^{\infty} D_nP_n^1(\cos\theta), \tag{3.4–3}$$

where

$$C_n=\frac{2n+1}{(n+1)K+n}\left[-na^{n-1}V_n+\frac{\gamma}{4\pi}(n+1)\frac{M_n}{a^{n+2}}\right], \tag{3.4–4}$$

$$D_n=\frac{2n+1}{(n+1)K+n}\left[Ka^{n-1}V_n+\frac{\gamma}{4\pi}\frac{M_n}{a^{n+2}}\right]. \tag{3.4–5}$$

Hence,

$$p_r=p_0+\frac{K-1}{2\gamma}\sum_{n=0}^{\infty}\sum_{s=0}^{\infty}[KC_nC_sP_n(\cos\theta)P_s(\cos_s\theta)+D_nD_sP_n^1(\cos\theta)P_s^1(\cos\theta)]. \tag{3.4–6}$$

The fourth step is the evaluation of the short-range force contributed by p_r. The z-component of force on an element $dS=2\pi a^2\sin\theta d\theta$ is

$$dF_z=-p_r\cos\theta dS=-p_r2\pi a^2\sin\theta\cos\theta d\theta. \tag{3.4–7}$$

Hence,

$$F_z=-2\pi a^2\int^{\pi}p_r\sin\theta\cos\theta d\theta$$

$$=-2\pi a^2\int_{-1}^{1}p_r\lambda d\lambda, \tag{3.4–8}$$

where

$$\lambda=\cos\theta. \tag{3.4–9}$$

Since the hydrostatic term p_0 contributes nothing to F_z,

$$F_z=-\frac{\pi}{\gamma}(K-1)a^2\sum_{n=0}^{\infty}\sum_{s=0}^{\infty}(KC_nC_sI_{ns}+D_nD_sJ_{ns}), \tag{3.4–10}$$

where

$$I_{ns}=\int_{-1}^{1}\lambda P_n(\lambda)P_s(\lambda)d\lambda, \tag{3.4–11}$$

$$J_{ns}=\int_{-1}^{1}\lambda P_n^1(\lambda)P_s^1(\lambda)d\lambda. \tag{3.4–12}$$

The recurrence formulas for the Legendre functions give

$$\lambda P_n=(2n+1)^{-1}[nP_{n-1}+(n+1)P_{n+1}], \tag{3.4–13}$$

$$\lambda P_n^1=(2n+1)^{-1}[(n+1)P^1{}_{n-1}+nP^1{}_{n+1}]; \tag{3.4–14}$$

it follows from the orthogonality relations that the only nonvanishing I's and J's are those for which the subscripts differ by 1. Each such term $(t, t+1)$ occurs twice: once for $n=t$, $s=t+1$ and once for $s=t$, $n=t+1$. Consequently,

$$F_z=-\frac{2\pi}{\gamma}(K-1)a^2\sum_{n=0}^{\infty}(KC_nC_{n+1}I_{n,n+1}+D_nD_{n+1}J_{n,n+1}), \tag{3.4–15}$$

with

$$I_{n,n+1}=\frac{n+1}{2n+1}\int_{-1}^{1}[P_{n+1}(\lambda)]^2d\lambda=\frac{2(n+1)}{(2n+1)(2n+3)}, \tag{3.4–16}$$

$$J_{n,n+1}=\frac{n}{2n+1}\int_{-1}^{1}[P_{n+1}^1(\lambda)]^2d\lambda=\frac{2n(n+1)(n+2)}{(2n+1)(2n+3)}; \tag{3.4–17}$$

or

$$F_2=-\frac{4\pi}{\gamma}(K-1)a^2\sum_{n=0}^{\infty}\frac{n+1}{(2n+1)(2n+3)}\times[KC_nC_{n+1}+n(n+2)D_nD_{n+1}]. \quad (3.4\text{–}18)$$

When the values of the C's and D's are substituted in this formula, the terms in V_nM_{n+1} cancel. The result is

$$F_2=\sum_{n=0}^{\infty}\left[-\frac{4\pi}{\gamma}\frac{n(n+1)K(K-1)}{(n+1)K+n}a^{2n+1}V_nV_{n+1}+\frac{(n+1)K(K-1)}{[(n+1)K+n][(n+2)K+(n+1)]}V_{n+1}M_n-\frac{\gamma}{4\pi}\frac{(n+1)(n+2)(K-1)}{(n+2)K+(n+1)}\frac{M_nM_{n+1}}{a^{2n+3}}\right]. \quad (3.4\text{–}19)$$

3.5 The Total Force in Various Cases

Finally, the sum of Eqs. (3.3–4) and (3.4–19) gives the total force. The terms in M_nM_{n+1} cancel, as was expected, and

$$F=\sum_{n=0}^{\infty}\left[-\frac{4\pi}{\gamma}\frac{n(n+1)K(K-1)}{(n+1)K+n}a^{2n+1}V_nV_{n+1}-\frac{(n+1)(2n+1)K}{(n+1)K+n}V_{n+1}M_n\right]. \quad (3.5\text{–}1)$$

The calculation of the total force by this method is based on the view that electric forces act directly at a distance and that the "stress" type of action, which takes place across surfaces of separation between adjacent volumes, is limited to regions occupied by matter. According to this view, the "long-range" and "short-range" forces F_1 and F_2 have physical significance—apart from a certain arbitrariness that was necessary (see Secs. 1.1–1.2) because it is impossible, in a macroscopic theory, to identify the two parts of the force of interaction when the interacting particles are molecularly near each other. The calculations of Stratton[3] are based on the alternative, maxwellian view that only the "stress" type of action occurs, that the presence of matter is not necessary for its occurrence, and that electric forces themselves are transmitted in this manner. From such a point of view, the separate forces F_1 and F_2 have no meaning, and the starting point must be Eq. (2.4–6). If the values of E_r and E_θ from Eqs. (3.4–2) and (3.4–3) are inserted in this equation and the integrations are carried out, by methods similar to those used in Sec. 3.4, the result, Eq. (3.5–1), will be obtained directly. In this manner, adherents of the maxwellian view may satisfy themselves of the correctness of the result; but they will have to provide their own discussion of it to replace the following discussion, parts of which will appear to them to be meaningless.

The force is made up of terms of two types. The first type, involving products V_nV_{n+1}, is independent of the charge distribution and is due entirely to fluid pressure on the sphere; these terms may be called bubble forces, for they are present even if the sphere is nothing but a bubble of un-ionized gas of negligible electric susceptibility. The second type, involving products $V_{n+1}M_n$, measures the force exerted by the external sources and the dielectric on the sphere in consequence of its charged state; these forces are due partly to fluid pressure and partly to direct electrical action. They may be called field-charge forces.

Of the field-charge terms, the leading two ($n=0$ and 1) are

$$F'_{[0]}+F'_{[1]}=-V_1M_0-6K(2K+1)^{-1}V_2M_1. \quad (3.5\text{–}2)$$

The potential $V_1rP_1(\cos\theta)=V_1z$ represents a uniform electric intensity $E_z'^0=-V_1$ along z; the potential $V_2r^2P_2(\cos\theta)$ gives $E_z'=-2V_2r\cdot P_1(\cos\theta)=-2V_2z$ and therefore represents a uniform gradient $g=-2V_2$ of E_z' in the z-direction. Hence,

$$F'_{[0]}+F'_{[1]}=qE_z'^0+3K(2K+1)^{-1}p_zg; \quad (3.5\text{–}3)$$

$E_z'^0$ and g are the external electric intensity and gradient at $(0, 0, 0)$. The force due to $E_z'^0$ is (for given E_z') independent of the dielectric constant of the fluid, but the force due to the gradient is not.

Of the bubble force terms, the one with $n=0$ vanishes. The one with $n=1$ is

$$F_{[1]}''=-\frac{4\pi}{\gamma}\frac{2K(K-1)}{2K+1}a^3V_1V_2=-\frac{4\pi}{\gamma}\frac{K(K-1)}{2K+1}a^3E_z'^0g. \quad (3.5\text{–}4)$$

One may say that a bubble in an electric field acquires a negative dipole moment proportional to the uniform component of the intensity (since at its surface the polarization of the surrounding dielectric has a normal component proportional to $\cos\theta$) and that this dipole moment is subject to a force proportional to the gradient of the intensity. However, it would be dangerous to attempt to evaluate the coefficient in the force by this type of reasoning.

The special case of a *conducting sphere* is easily treated. In this case, the moments M_n, which are due to charges on the surface, must have such values that the potential becomes a constant at $r=a$. By Eq. (3.2–11), this requires

$$M_n=-(4\pi/\gamma)Ka^{2n+1}V_n \quad \text{for} \quad n>0, \quad (3.5\text{–}5)$$

and the force is

$$F=-V_1M_0+\frac{4\pi}{\gamma}\sum_{n=1}^{\infty}(n+1)Ka^{2n+1}V_nV_{n+1}$$

$$=qE_z'^0+\frac{4\pi}{\gamma}\sum_{n=1}^{\infty}(n+1)Ka^{2n+1}V_nV_{n+1}. \quad (3.5\text{–}6)$$

Each term in the sum is partly a bubble force and partly a field-charge force. The gradient term is $(4\pi/\gamma)Ka^3E_z'^0g$.

The special case of a *dielectric sphere*, with dielectric constant K_1, is slightly more complicated. Here the moments are due to surface polarization charges. The surface charge density due to the polarization *of the sphere* can be expanded in the form

$$\sigma(\theta)=\sum_{s=1}^{\infty}\sigma_sP_s(\cos\theta), \quad (3.5\text{–}7)$$

and the moment M_n is then given by

$$M_n=a^n\int\sigma(\theta')P_n(\cos\theta')dS'$$

$$=2\pi a^{n+2}\int_0^{\pi}\sigma(\theta')P_n(\cos\theta')\sin\theta'd\theta'$$

$$=[4\pi\sigma_n/(2n+1)]a^{n+2}. \quad (3.5\text{–}8)$$

In Eq. (3.2–4), the integral from 0 to r vanishes and the integral from r to a degenerates to $\sigma(\theta')/a^{n-1}$. When the integrations in Eq. (3.2–4) are carried out and the result is substituted in Eqs. (3.2–3) and (3.2–10), there results an expression for $V_{(1)}$ involving the M_n's and σ_n's; the M_n's may be eliminated by use of Eq. (3.5–8). The procedure for evaluating the σ_n's is as follows: differentiate $V_{(1)}$ to find E_r; multiply by $(K_1-1)/\gamma$ to find P_r; set the value of P_r at $r=a$ equal to σ; equate coefficients of $P_n(\cos\theta)$; and solve for σ_n. Equation (3.5–8) then gives

$$M_n=-\frac{4\pi}{\gamma}\frac{nK(K_1-1)V_na^{2n+1}}{(n+1)K+nK_1}, \quad (3.5\text{–}9)$$

which reduces to Eq. (3.5–5) for $K_1=\infty$ and vanishes for $K_1=1$. Substitution of this value of M_n in the force formula (3.5–1) gives

$$F=-\frac{4\pi}{\gamma}\sum_{n=1}^{\infty}\frac{n(n+1)K(K-K_1)a^{2n+1}V_nV_{n+1}}{(n+1)K+nK_1}. \quad (3.5\text{–}10)$$

This vanishes for $K_1=K$; reduces to the conductor formula (3.5–6) (with $q=0$) for $K_1=\infty$; and reduces to the bubble force part of Eq. (3.5–1) for $K_1=1$. The leading term ($n=1$) is

$$F_{[1]}=-\frac{4\pi}{\gamma}\frac{K(K-K_1)}{2K+K_1}a^3E_z'^0g, \quad (3.5\text{–}11)$$

which includes the previous formulas for the bubble and for the conducting sphere.

From the symmetry of the field and charges, there can be no resultant torque about the center of the sphere in the case considered. The problem will now be altered slightly in order to consider one special case in which there is a torque. Suppose that the charge distribution is, as before, symmetric about the z axis, but that the external electric intensity is uniform and in the direction of the x axis. Then the electric field strength produced in the sphere by sources outside contains, in addition to the previous terms in the M_n's (which by symmetry contribute no torque), a uniform x-component

$$E_{0x}=3K(2K+1)^{-1}E_x', \quad (3.5\text{–}12)$$

where E_x' is the external intensity. The long-range torque L_y about the y axis is found by multiplying Eq. (3.5–12) by $\rho z d\tau$ and integrating, so that

$$L_y=3K(2K+1)^{-1}E_x'\int\rho z d\tau$$

$$=3K(2K+1)^{-1}p_zE_x'. \quad (3.5\text{–}13)$$

The torque due to fluid pressure vanishes for a sphere, since all the forces are directed toward the center; therefore, Eq. (3.5–13) also gives the resultant torque. The factor that occurs in the torque formula is the same factor that occurs in the dipole force term in Eq. (3.5–3). The potential produced outside the sphere by the dipole moment is, by Eq. (3.2–11),

$$V_{(2)[1]}=\frac{\gamma}{4\pi}\frac{3M_1}{2K+1}\frac{P_1(\cos\theta)}{r^2}$$

$$=\frac{\gamma}{4\pi}\frac{3}{2K+1}\frac{p_z\cos\theta}{r^2}. \quad (3.5\text{–}14)$$

If one naively (and, of course, erroneously) applies to a permanently polarized sphere the theorem that the intensity is decreased in the ratio $1/K$ by immersion in a polarizable fluid and that the relation of force to intensity is not changed, one obtains formulas for the external potential due to the dipole moment and for the force and torque on it, all of which are in error by the same factor: all must be multiplied by $3K/(2K+1)$ to get the right value. This is in accordance with the previous conclusions of Wilberforce[9] and of Page[10] and with Eq. (2.5–9) (here, $\kappa_m'=1$, $D=\frac{1}{3}$).

The general form of the terms in the force and potential shows clearly that the case in which the factors are as simple as 1 and $1/K$ is an exceptional one. The ratio of the coefficient of $V_{n+1}M_n$ in Eq. (3.5–1) to its value for $K=1$ is $(2n+1)K/[(n+1)K+n]$, which is unity only for $n=0$. The ratio of the coefficient of M_n in Eq. (3.2–11) to its value for $K=1$ is $(2n+1)/[(n+1)K+n]$, which is $1/K$ only for $n=0$. The $1/K$ theorem applies, of course, when the fluid fills all parts of space where an electric field exists. It applies, therefore, to the conducting sphere, if the external field is itself produced by charges on insulated conductors. Insertion of Eq. (3.5–5) in Eq. (3.2–11) gives for the conducting sphere

$$V_{(2)}=\sum_{n=0}^{\infty}V_n r^n P_n(\cos\theta)+\frac{\gamma}{4\pi}\frac{q}{Kr}$$

$$-\sum_{n=1}^{\infty}a^{2n+1}V_n\frac{P_n(\cos\theta)}{r^{n+1}}. \quad (3.5\text{–}15)$$

If the entire system of insulated conductors, initially in a vacuum, is immersed in the fluid, the V_n's are all decreased in the ratio $1/K$, and, therefore, all terms in $V_{(2)}$ are decreased in that ratio. In the force formula (3.5–6), $E_z'^0$ and all the V_n's are decreased in the ratio $1/K$; the formula shows that F is then decreased in the same ratio.

The example considered is sufficiently general to illustrate the principles involved. The consideration of a charge distribution and field that are not symmetric about the z axis is left as an exercise for the ambitious reader. The still more ambitious reader may carry out the corresponding calculations for prolate and oblate spheroids.

The electric case has been treated here in order to include the M_0 term. In the magnetic case, $M_0=0$; otherwise, the *parallel* analysis by use of the scalar potential is identical. However, the magnetic problem completely analogous—in the *antiparallel* sense—to the present one is somewhat more general, namely, the problem of a sphere carrying electric currents (conduction currents or amperian currents or both) in circles about the z axis. This problem could be handled by use of the vector potential precisely as the present one was handled by use of the scalar potential; and the case of a permeable sphere (with constant permeability) could be derived as a special case of currents, just as the case of a dielectric sphere was derived as a special case of charges. In the absence of conduction currents or charges, the electric and magnetic problems can each be treated either by use of polarization charges and a scalar potential or by use of polarization currents and a vector potential. The use of currents in the electric case is no more artificial than the use of charges (poles) in the magnetic.[14]

4. ENERGY, THERMODYNAMIC FUNCTIONS, AND THE ELASTIC SOLID

4.1 Program and Precautions

In this analysis of electric and magnetic forces, the guiding motive has been to derive all results, as far as was possible, by direct calculation of forces and without introduction of special assumptions. In the case of fluids, it proved

[14] William Fuller Brown, Jr., Am. J. Phys. **8**, 338–345 (1940).

possible to derive all the standard results from the definition of a fluid, from the equilibrium condition, and from the further definition of an important class of fluids—those for which the ratio of polarization to field intensity is (at a given temperature) a function only of the density. The relations thus obtained constitute the generalization, for a polarizable fluid in an electric field, of the equilibrium relations for an unpolarized fluid. The object in Part 4 is to carry out the corresponding generalization for a solid body. It will be helpful to begin by comparing the fluid and the solid when each is unpolarized.

For an unpolarized fluid, $M=0$, Eq. (2.2–2) becomes

$$\int_{\rho 0}^{\rho}(dp/\rho)+V=\text{const}; \qquad (4.1\text{–}1)$$

and this determines (once the pressure-density relation is known) the variation of pressure with position in a conservative field of force. In particular, it shows that when no body forces act, the pressure must be uniform throughout the container. The relation (4.1–1) requires no energy argument for its derivation, but it may be given an energy interpretation by considering a particle that moves in a leisurely fashion (i.e., with negligible kinetic energy) from one point to another.

The properties of unpolarized solids are more complicated than those of unpolarized fluids. Instead of a single p-ρ or p-V relation, there is a system of relations between six stress components and six strain components, and the equations of equilibrium have no such simple integral as Eq. (4.1–1). In order to integrate the equations, it is necessary to introduce explicit formulas for the stress-strain relations: linear ones for an elastic solid, more complicated ones for a plastic solid. To simplify the elastic relations still further, it is usual to assume that the elastic processes are reversible; this reduces the number of elastic constants, thanks to reciprocal relations imposed by thermodynamics. These various relations are not equivalent to ones that can be obtained by comparing particles at different points, because the particles are not free to travel about as in a fluid.

The program for Part 4 is to extend this treatment to the case of a polarizable solid. To do so it is necessary—as in the case of unpolarized solids—to introduce explicit formulas and, further, to invoke thermodynamic arguments (based on the reversibility postulate) in order to reduce the number of coefficients. This will be done, as far as is possible, without introducing any special assumptions about the magnetization curve—though, of course, the reversibility postulate is itself a serious restriction, in view of the importance of hysteresis in ferromagnetic materials.

The role of energy arguments in this treatment is much less basic than in the conventional treatments. There energy is used to derive the equilibrium equations themselves; here it is used only to supplement them by introducing simple and plausible formulas to describe the properties of special classes of matter. Therefore, the stringent rules set up at the beginning—that no special assumptions, other than those that define the class of matter to be considered, are to be introduced and that no significance is to be attributed in advance to the pole and amperian current densities of the macroscopic theory or to the macroscopic field vectors—will not be relaxed. In fact, they must become more stringent in one respect.

In Part 1, where only forces and torques were being computed, the alternative interpretations in terms of polarization charge density and polarization current density were treated with considerable tolerance, because they led to results equivalent to those obtained by the dipole method. Here it is necessary to be less tolerant. In calculating the force and torque on a uniformly electrically polarized spherical portion of a fluid, it is immaterial whether the long-range electric forces are supposed to act on the dipoles or on the charges of density $-\nabla\cdot\mathbf{P}$, for though the first method gives an electric torque and the second gives none, the difference is made up in the stresses, which by the second method include the forces exerted on charges just outside the boundary and transmitted across it mechanically (Sec. 2.1). But in calculating the work done in a small displacement or deformation, the localization of the force is more important. If it is supposed to act at the surface of the region when it actually acts in its interior, there is no

guarantee that the work will be correctly evaluated.

The basic point of view adopted here is that the long-range forces act on the dipole moments. In the case of the spherical region, this entails an ambiguity only with respect to the matter molecularly near the boundary of the region: there, as was remarked in Sec. 1.1, the separation of the total force into a long-range force and a stress is non-unique in any purely macroscopic theory. The convention adopted here is that the magnetic or "long-range" force is to be computed by the use of continuous dipole densities, and whatever is left over is to be called "stress." This convention is arbitrary but will not affect the result of a macroscopic energy calculation, because localization of the force "at" the boundary or molecularly "inside" it amounts to the same thing on a macroscopic scale.

The program, then, is to compute the rate of work by long-range forces and by tractions on that matter which, at the instant under consideration, occupies a volume τ; to identify parts of this work that can be recognized as going into kinetic energy or potential energy of known forms; and to ascertain whether the remainder is of such form that it can be asserted with certainty to have been either dissipated or stored by mechanisms internal to τ. If so, the thermodynamics of reversible processes can then be used to simplify the magnetoelastic relations for materials with negligible hysteresis.

4.2 Work and Energy Rates

The calculation of the rate of work by long-range magnetic forces requires some care. Consider first a small, ideally permanent magnet of moment $\mathbf{m}$ in a field $\mathbf{H}_0$ that is constant in time. The force and couple, $\mathbf{m}\cdot\nabla\mathbf{H}_0$ and $\mathbf{m}\times\mathbf{H}_0$, can be calculated by differentiating a potential energy $-\mathbf{m}\cdot\mathbf{H}_0$ with respect to the translational and angular coordinates of the magnet; and if the magnet moves with velocity $\mathbf{v}$ and angular velocity $\boldsymbol{\Omega}$, the rate of work by the force and couple is equal to the rate of decrease of the potential energy. Suppose now that the magnet is not permanent, but that its moment changes at a rate $D\mathbf{m}/dt$ with respect to axes that translate *and rotate* with the magnet. Then, with respect to axes that translate *but do not rotate* with the magnet, the rate of change of $\mathbf{m}$ is

$$d\mathbf{m}/dt=\boldsymbol{\Omega}\times\mathbf{m}+D\mathbf{m}/dt, \qquad (4.2\text{–}1)$$

whereas previously only the first term was present. The rate of change of the field strength acting on the magnet is $\mathbf{v}\cdot\nabla\mathbf{H}_0$. The rate of decrease of potential energy is

$$-\frac{d}{dt}(-\mathbf{m}\cdot\mathbf{H}_0)=\mathbf{m}\cdot\frac{d\mathbf{H}_0}{dt}+\frac{d\mathbf{m}}{dt}\cdot\mathbf{H}_0=\mathbf{m}\cdot[\mathbf{v}\cdot\nabla\mathbf{H}_0]$$
$$+\boldsymbol{\Omega}\times\mathbf{m}\cdot\mathbf{H}_0+(D\mathbf{m}/dt)\cdot\mathbf{H}_0=\mathbf{v}\cdot[\mathbf{m}\cdot\nabla\mathbf{H}_0]$$
$$+\boldsymbol{\Omega}\cdot[\mathbf{m}\times\mathbf{H}_0]+\mathbf{H}_0\cdot\frac{D\mathbf{m}}{dt}. \qquad (4.2\text{–}2)$$

(The interchange of $\mathbf{m}$ and $\mathbf{v}$ in the first term is permissible because $\nabla\times\mathbf{H}_0=0$.) The first two terms evaluate the rate of work by the force $\mathbf{m}\cdot\nabla\mathbf{H}_0$ and the couple $\mathbf{m}\times\mathbf{H}_0$. Besides these, there is a term $\mathbf{H}_0\cdot D\mathbf{m}/dt$ due to changes internal to the magnet. If the magnet is to be treated as a conservative dynamical system, it is necessary to introduce, besides the external coordinates necessary for the permanent magnet, additional internal coordinates—the components of $\mathbf{m}$ in axes attached to the magnet—and to include, in the differential expression for work done, a term $\mathbf{H}_0\cdot D\mathbf{m}$.

A direct physical interpretation of these coordinates is possible in the simple case of a single electrically polarizable molecule: the external coordinates describe the position and orientation of the massive part of the molecule, the internal coordinates (components of electric moment in axes attached to the molecule) describe the displacements in these axes of the electrons responsible for the moment. In a macroscopic theory it is not necessary to resort to such an interpretation, but it is necessary to introduce the additional degrees of freedom and to include their contribution to the work. Introduction of additional coordinates implies that additional terms in the kinetic energy may also be needed. This is, in fact, the case, but the kinetic terms are important only at high frequencies (at least when the susceptibility is positive); they will be neglected here.

If $\mathbf{H}_0$ is itself produced by a magnet, the additional term $\mathbf{m}\cdot\partial\mathbf{H}_0/\partial t$—where $\partial/\partial t$ denotes rate of change at a fixed point—can be shown

without difficulty to be equal to the rate of work on the second magnet. In fact, the mutual potential energy of two small magnets $\mathbf{m}_1, \mathbf{m}_2$, separated by vector distance $\mathbf{r}=\mathbf{r}_1-\mathbf{r}_2$, is $\mathbf{m}_1\cdot\mathbf{m}_2/r^3-3\mathbf{m}_1\cdot\mathbf{r}\ \mathbf{m}_2\cdot\mathbf{r}/r^5$; the terms in the rate of change that are due to change of $\mathbf{m}_1$ and of $\mathbf{r}_1$ give Eq. (4.2–2) written for magnet 1, and those that are due to change of $\mathbf{m}_2$ and of $\mathbf{r}_2$ give Eq. (4.2–2) written for magnet 2. Therefore, even if the field $\mathbf{H}_0$ is not constant, the expression after the last equality sign in Eq. (4.2–2) still gives the rate of work on magnet $\mathbf{m}$ by long-range magnetic forces exerted by the sources of the field.

For a definite body of matter, of mass m, forming part of a larger magnetized body and instantaneously occupying a volume τ, $\mathbf{m}$ is to be replaced by $\mathbf{M}d\tau=\mathbf{M}dm$, where dm is an element of mass, and the result is to be integrated over τ. The rate of work on m by long-range magnetic forces exerted by sources outside m is therefore

$$\frac{dW_m}{dt}=\int\left\{\mathbf{v}\cdot[\mathbf{M}\cdot\nabla\mathbf{H}_0]+\boldsymbol{\Omega}\cdot[\mathbf{M}\times\mathbf{H}_0]+\mathbf{H}_0\cdot\frac{D\mathbf{M}}{dt}\right\}dm=\int\{\mathbf{v}\cdot[\mathbf{M}\cdot\nabla\mathbf{H}_0]+\boldsymbol{\Omega}\cdot[\mathbf{M}\times\mathbf{H}_0]\}d\tau+\int\mathbf{H}_0\cdot\frac{D\mathbf{M}}{dt}dm. \quad (4.2\text{–}3)$$

This work is done against inertial, internal, or body forces in the mass elements; against mutual forces (magnetic or mechanical) between them; and against tractions exerted across the boundary surface.

The rate of work by the tractions is (if $\mathbf{v}$ has components v_x, v_y, v_z)

$$\begin{aligned}\frac{dW_t}{dt}&=\int\{X_xv_x+Y_xv_y+Z_xv_z\}dS\\&=\int\{(X_x+\tfrac{1}{2}\gamma M_x^2-\tfrac{1}{2}\gamma M_n^2)v_x\\&\quad+X_yv_xm+X_zv_xn+\cdots\}dS\\&=\int\left\{v_x\left[\frac{\partial}{\partial x}(X_x+\tfrac{1}{2}\gamma M_x^2)+\frac{\partial X_y}{\partial y}+\frac{\partial X_z}{\partial z}\right]\right.\\&\quad\left.+\cdots\right\}d\tau-\tfrac{1}{2}\gamma\int M_n^2\mathbf{n}\cdot\mathbf{v}dS\\&\quad+\int\left\{(X_x+\tfrac{1}{2}\gamma M_x^2)\frac{\partial v_x}{\partial x}+X_y\frac{\partial v_x}{\partial y}\right.\\&\quad\left.+X_z\frac{\partial v_x}{\partial z}+\cdots\right\}d\tau. \quad (4.2\text{–}4)\end{aligned}$$

Now the velocity strains are $\epsilon_{xx}=\partial v_x/\partial x, \cdots, \epsilon_{xy}=(\partial v_y/\partial x)+\partial v_x/\partial y, \cdots$ (for small elastic strains these are the time derivatives of the usual elastic "strains"), and the angular velocity is $\boldsymbol{\Omega}=\frac{1}{2}\nabla\times\mathbf{v}$; thus, $\partial v_y/\partial x=\frac{1}{2}\epsilon_{xy}+\Omega_z$, $\partial v_x/\partial y=\frac{1}{2}\epsilon_{xy}-\Omega_z$, etc. When the last integral in Eq. (4.2–4) is expressed in terms of these quantities and use is made of the values of Y_z-Z_y, etc., from Eq. (1.5–1), the integral becomes

$$\int\{(X_x+\tfrac{1}{2}\gamma M_x^2)\epsilon_{xx}+\cdots+\tfrac{1}{2}(X_y+Y_x)\epsilon_{xy}+\cdots-\mathbf{M}\times\mathbf{H}\cdot\boldsymbol{\Omega}\}d\tau. \quad (4.2\text{–}5)$$

The sum of Eqs. (4.2–3) and (4.2–4) gives the rate of work by long-range forces and tractions together on the matter in τ. The mechanical body forces do work at the rate $\int\rho(Xv_x+\cdots)d\tau$; and when all these rates of work are added together, some of the result is accounted for as kinetic energy, which increases at the rate $\int\rho(f_xv_x+\cdots)d\tau$. By Eq. (1.5–3), the rate of work by body forces minus the rate of increase of kinetic energy is

$$\begin{aligned}\frac{dW_b}{dt}-\frac{dT}{dt}&=\int\rho[(X-f_x)v_x+\cdots]d\tau\\&=-\int\left\{v_x\left[\frac{\partial}{\partial x}(X_x+\tfrac{1}{2}\gamma M_x^2)+\frac{\partial X_y}{\partial y}+\frac{\partial X_z}{\partial z}\right]\right.\\&\quad\left.+\cdots+\mathbf{v}\cdot(\mathbf{M}\cdot\nabla\mathbf{H})\right\}d\tau. \quad (4.2\text{–}6)\end{aligned}$$

When this is added to the sum of Eqs. (4.2–3) and (4.2–4), with use of Eq. (4.2–5), the result is

$$\begin{aligned}\frac{dW_m}{dt}+\frac{dW_t}{dt}+\frac{dW_b}{dt}-\frac{dT}{dt}&=\int\mathbf{H}_0\cdot\frac{D\mathbf{M}}{dt}dm\\&\quad-\int[\boldsymbol{\Omega}\cdot\mathbf{M}\times\mathbf{H}_1+\mathbf{v}\cdot(\mathbf{M}\cdot\nabla\mathbf{H}_1)]d\tau\\&\quad-\tfrac{1}{2}\gamma\int M_n^2\mathbf{n}\cdot\mathbf{v}dS+\int\{(X_x+\tfrac{1}{2}\gamma M_x^2)\epsilon_{xx}\\&\quad+\cdots+\tfrac{1}{2}(X_y+Y_x)\epsilon_{xy}+\cdots\}d\tau, \quad (4.2\text{–}7)\end{aligned}$$

where $\mathbf{H}_1=\mathbf{H}-\mathbf{H}_0$ is the part of the magnetic intensity that is due to the magnetization in τ.

Without introducing an atomic model, it is impossible to say how much of the power (4.2–7) goes into mutual magnetic energy of the atomic magnets in τ and how much into mechanical internal energy and heat. Nevertheless, it is convenient to separate Eq. (4.2–7) into two terms, one of which is formally a sort of magnetic energy. By methods similar to those of Sec. 1.4, it may be shown that the quantity (see Fig. 1, Part 1)

$$U_m=-\frac{1}{2}\int_\tau \mathbf{M}\cdot\mathbf{H}_1 d\tau = \lim_{S_1,S_2\to S}(-\tfrac{1}{2})\int_{\tau_2}\mathbf{M}_2\cdot\mathbf{H}_{12}d\tau_2 \quad (4.2\text{–}8)$$

changes (for a given body of matter, occupying a perhaps variable region τ) at the rate

$$\frac{dU_m}{dt}=-\int\mathbf{H}_1\cdot\frac{D\mathbf{M}}{dt}dm - \int[\boldsymbol{\Omega}\cdot\mathbf{M}\times\mathbf{H}_1+\mathbf{v}\cdot(\mathbf{M}\cdot\nabla\mathbf{H}_1)]d\tau - \tfrac{1}{2}\gamma\int M_n^2\mathbf{n}\cdot\mathbf{v}dS, \quad (4.2\text{–}9)$$

and therefore

$$\frac{dW_m}{dt}+\frac{dW_t}{dt}+\frac{dW_b}{dt}-\frac{dT}{dt}=\frac{dU_m}{dt}+\int\mathbf{H}\cdot\frac{D\mathbf{M}}{dt}dm + \int\{(X_x+\tfrac{1}{2}\gamma M_x^2)\epsilon_{xx}+\cdots + \tfrac{1}{2}(X_y+Y_x)\epsilon_{xy}+\cdots\}d\tau. \quad (4.2\text{–}10)$$

The derivation of Eq. (4.2–9) is given in Appendix A.

A similar procedure based on the **B** equations leads to a different separation of the right member into two terms. Let

$$U_m'=-\tfrac{1}{2}\int_\tau\mathbf{M}\cdot\mathbf{B}_1 d\tau = \lim_{S_1,S_2\to S}(-\tfrac{1}{2})\int\mathbf{M}_2\cdot\mathbf{B}_{12}d\tau_2: \quad (4.2\text{–}8')$$

then

$$\frac{dW_m}{dt}+\frac{dW_t}{dt}+\frac{dW_b}{dt}-\frac{dT}{dt}=\frac{dU_m'}{dt}+\int\mathbf{B}\cdot\frac{D\mathbf{M}}{dt}dm + \int\{[X_x-\tfrac{1}{2}\gamma(M_y^2+M_z^2)]\epsilon_{xx}+\cdots + \tfrac{1}{2}(X_y+Y_x)\epsilon_{xy}+\cdots\}d\tau. \quad (4.2\text{–}10')$$

4.3 Thermodynamic Functions in General and for a Fluid

The advantage of separating the work rate into terms as was done in Eqs. (4.2–10) and (4.2–10′) is that the part that depends on the shape of m is entirely contained in the term dU_m/dt or dU_m'/dt, and this is the rate of change of a quantity uniquely determined by the magnetization and strains in m. It follows that the remaining terms must together, in any reversible isothermal or adiabatic change, be equal to the rate of change of a quantity determined by the magnetization and strains in m and must, when integrated over a complete cycle of a periodic irreversible process, measure the energy dissipated in the mass m. For definiteness the changes will be supposed to be isothermal. Then, if

$$dF\equiv\mathbf{H}\cdot D\mathbf{M}+V\{(X_x+\tfrac{1}{2}\gamma M_x^2)de_{xx}+\cdots + \tfrac{1}{2}(X_y+Y_x)de_{xy}+\cdots\}, \quad (4.3\text{–}1)$$

$$dF'\equiv\mathbf{B}\cdot D\mathbf{M}+V\{[X_x-\tfrac{1}{2}\gamma(M_y^2+M_z^2)]de_{xx} + \cdots+\tfrac{1}{2}(X_y+Y_x)de_{xy}+\cdots\} \quad (4.3\text{–}1')$$

(in which $de_{xx}\equiv\epsilon_{xx}dt$, etc.), $\int_m dF dm$ and $\int_m dF' dm$ are perfect differentials in a reversible process, and their integrals may be used as thermodynamic potentials (free energies) for the mass m. Moreover, this holds with respect to an arbitrary mass, and the value of $\int dF dm$ or $\int dF' dm$ for a mass m_1+m_2 is the sum of its values for m_1 and m_2; therefore, dF and dF' themselves are perfect differentials, and their integrals F and F' may be used as thermodynamic potentials for unit mass. The relation between these two functions is

$$F'-F=\tfrac{1}{2}\gamma M^2V=\tfrac{1}{2}\gamma M^2/V. \quad (4.3\text{–}2)$$

The equivalence of Eqs. (4.3–1) and (4.3–1′) is easily established by means of Eq. (4.3–2). If $d\boldsymbol{\omega}\equiv\boldsymbol{\Omega}dt$, then $d(M^2)=2\mathbf{M}\cdot d\mathbf{M}=2\mathbf{M}\cdot(D\mathbf{M}+d\boldsymbol{\omega}\times\mathbf{M})=2\mathbf{M}\cdot D\mathbf{M}$; and $d(V^{-1})=-V^{-1}(\nabla\cdot\mathbf{v})dt$

$= -V^{-1}(de_{xx}+de_{yy}+de_{zz})$. Therefore, from Eq. (4.3–2)

$$d(F'-F) = \tfrac{1}{2}\gamma V^{-1}d(M^2)+\tfrac{1}{2}\gamma M^2 d(V^{-1}) = \gamma \mathbf{M}\cdot D\mathbf{M} - \tfrac{1}{2}\gamma M^2 V(de_{xx}+de_{yy}+de_{zz}); \quad (4.3\text{–}3)$$

and when Eq. (4.3–3) is added to Eq. (4.3–1), the result is Eq. (4.3–1′).

Several points are worth emphasizing in regard to these functions F and F'. (1) The derivation given avoids the usual unjustifiable step by which an integral *over all space* is used as the basis for an expression alleged to represent energy associated with an element of volume. The corresponding step here is from an integral *over an arbitrary mass* to an expression related to unit mass. (2) Neither function, $\int F dm$ or $\int F' dm$, represents *all* the free energy associated with the mass m; and neither function represents a part of the free energy that necessarily has any simple physical significance, for the functions U_m and U_m', though formally interpretable as mutual potential energy of poles and of amperian currents, respectively, have no necessary significance as actual mutual energies of atomic magnets. All that can be asserted is that the functions F and F', being free of shape dependence, are suitable for use as specific thermodynamic potentials. If the whole energy is to be calculated, the shape-dependent term U_m or U_m' must be added. (3) If two adjoining macroscopic parts m_1, m_2 of a body are regarded as together constituting a part m, the relations $\int_m F dm = \int_{m_1} F dm + \int_{m_2} F dm$, $\int_m F' dm = \int_{m_1} F' dm + \int_{m_2} F' dm$ hold; but the value of U_m or U_m' for m differs from the sum of the separate values by a term equal to the mutual (long-range) magnetic energy of the two parts. (4) Any linear combination $\alpha\mathbf{H}+(1-\alpha)\mathbf{B}$ could be used instead of $\mathbf{H}$ or $\mathbf{B}$ (α=const, $0\leq\alpha\leq 1$) by removal of a shape-dependent energy term $\alpha U_m+(1-\alpha)U_m'$; the shape-dependent "magnetic" energy is therefore indeterminate, but the difference between any two values is shape-independent by Eq. (4.3–2).

For use of $\mathbf{H}$ or $\mathbf{B}$ rather than of $\mathbf{M}$ as independent variable, the functions

$$Z = F - \mathbf{H}\cdot\mathbf{M},\quad Z' = F' - \mathbf{B}\cdot\mathbf{M} \qquad (Z'-Z = -\tfrac{1}{2}\gamma M^2/V) \quad (4.3\text{–}4)$$

are more convenient. In the formulas for dZ and dZ', the first terms in Eqs. (4.3–1) and (4.3–1′) are replaced by $-\mathbf{M}\cdot D\mathbf{H}$ and $-\mathbf{M}\cdot D\mathbf{B}$, respectively. (Note that $D(\mathbf{M}\cdot\mathbf{H})$ and $d(\mathbf{M}\cdot\mathbf{H})$ mean the same thing, since $\mathbf{M}\cdot\mathbf{H}$ is a scalar.) For a fluid, the functions Z and Z' reduce to those of Sec. 2.2: in this case $X_y = Y_z = 0$, $X_x + \tfrac{1}{2}\gamma M_x^2 = -p'$, and $\mathbf{M}$ is along $\mathbf{H}$ (so that $\mathbf{M}\cdot D\mathbf{H} = (M/H)\mathbf{H}\cdot D\mathbf{H} = \tfrac{1}{2}(M/H)D(H^2) = M dH$), and therefore

$$dZ = -M dH - p'V(de_{xx}+de_{yy}+de_{zz}) = -M dH - p' dV. \quad (4.3\text{–}5)$$

The choice of $\mathbf{H}$, $\mathbf{B}$, or $\alpha\mathbf{H}+(1-\alpha)\mathbf{B}$ as the variable to be used is a matter of convenience only. No interpretation of any of these as an "effective" field intensity, and of the corresponding quantity U_m, U_m', or $\alpha U_m+(1-\alpha)U_m'$ as magnetic energy in a literal sense, is either justified by the analysis or necessary for the derivation of useful results.

If the changes are still reversible but include changes of absolute temperature T, a term $-S dT$ (where S is specific entropy) must be added to Eq. (4.3–1) and to Eq. (4.3–1′). If the original thermodynamic potential had been that corresponding to adiabatic changes, the additional term would be $+T dS$; this thermodynamic potential differs from F by TS and corresponds to choice of S rather than of T as independent variable. By choosing various combinations of variables as the independent ones and writing the conditions that the infinitesimal increments of the corresponding thermodynamic potentials shall be perfect differentials, one can derive a large number of relations of the sort familiar in traditional thermodynamics; for a fluid, for instance, it follows from Eq. (4.3–5) with the term $-S dT$ added that $(\partial M/\partial T)_{V,H} = (\partial S/\partial H)_{V,T}$. Relations of this type, however, are of very little use in discussing the behavior of solids at a given temperature. This will be the topic of the next section.

4.4 The Elastic Solid

For a solid material, consider a mass element dm; and let $(x^{(1)}, y^{(1)}, z^{(1)})$ refer to axes with origin at the mass-centroid of dm, and rotating with the angular velocity $\boldsymbol{\Omega} = \tfrac{1}{2}\nabla\times\mathbf{v}$ of the matter at the centroid. Then the first term in Eq. (4.2–9) is $H_x^{(1)}dM_x^{(1)}+H_y^{(1)}dM_y^{(1)}+H_z^{(1)}dM_z^{(1)}$. To transform the other terms to the $(x^{(1)}, y^{(1)}, z^{(1)})$

axes, transform first to axes $(x^{(2)}, y^{(2)}, z^{(2)})$ that coincide instantaneously with the $(x^{(1)}, y^{(1)}, z^{(1)})$ axes but do not rotate. The transformation law for X_x, etc., can be found without difficulty from Eq. (1.5–2) applied to the cases of normals along $x^{(2)}, y^{(2)}, z^{(2)}$ in turn; it is easily shown that the quantities

$$X_x^* \equiv X_x + \tfrac{1}{2}\gamma M_x^2, \cdots, \tag{4.4–1}$$

$$X_y^* \equiv Y_z^* \equiv \tfrac{1}{2}(X_y + Y_z), \cdots \tag{4.4–2}$$

obey the transformation laws which are obeyed by the stresses in unpolarized matter. The transformation law for $de_{xx} = \epsilon_{xx}dt = (\partial v_x/\partial x)dt$, etc., is not affected by the polarization. The expression $X_x^* de_{xx} + \cdots + X_y^* de_{xy} + \cdots$ therefore transforms as does $X_x de_{xx} + \cdots + X_y de_{xy} + \cdots$ in unpolarized matter; i.e., it becomes $X_x^{*(2)} de_{xx}^{(2)} + \cdots + X_y^{*(2)} de_{xy}^{(2)} + \cdots$. Now transform to the $(x^{(1)}, y^{(1)}, z^{(1)})$ axes: the stress factors are unaffected, and the addition of a rigid-body rotation about a fixed point does not affect the velocity strains. Equation (4.3–1) therefore becomes—with the superscripts now dropped—

$$dF = H_x dM_x + \cdots + V\{X_x^* de_{xx} + \cdots + X_y^* de_{xy} + \cdots\}; \tag{4.4–3}$$

the (x, y, z) axes now are attached to dm and rotate with it.

When this meaning is assigned to the symbols, M_x, M_y, and M_z describe a magnetic state of the matter in dm; and with the usual approximations of elasticity (which are introduced for the first time at this point), the quantities e_{xx}, e_{yy}, e_{zz}, e_{yz}, e_{zx}, e_{xy} describe a state of strain of the matter. For reversible behavior, the thermodynamic potential F may be taken to be a single-valued function of these nine variables. If it is so expressed, the other nine variables are then given by

$$H_x = \partial F/\partial M_x, \cdots, \quad VX_x^* = \partial F/\partial e_{xx}, \cdots, \quad VY_z^* = \partial F/\partial e_{yz}, \cdots. \tag{4.4–4}$$

In X_x^*, etc., the variation of the specific volume V with strain needs to be taken into account only in the terms that are of zeroth order in the strains. One Y_z^* is known, Y_z and Z_y can be found by means of Eqs. (1.5–1) and (4.4–2).

Only two cases will be discussed in greater detail and those only briefly.

(a) For many materials, it is sufficient to express F as a power series in all nine variables and to neglect terms of higher order than the second. Relations among the constant coefficients are then determined by the crystal symmetry. This procedure leads to formulas for H_x, H_y, H_z, X_x^*, Y_y^*, Z_z^*, Y_z^*, Z_x^*, X_y^* as linear functions of the magnetization components and strains. The relations are those that were worked out in detail by Voigt[15] in his analysis of the electric and magnetic polarizability, elasticity, and piezoelectricity and -magnetism of crystals. It will be noticed, however, that Voigt's stresses $-X_x$, $-X_y$, $\cdots$ are to be identified not with the present $X_x, X_y, \cdots$, but with the present $X_x^*, X_y^*, \cdots$; and that, according to the present analysis, Voigt's vector and tensor components must be interpreted as components in axes rotating with the matter.

Voigt himself began by developing the basic concepts and the expressions for incremental work separately for polarization without strain and for strain without polarization; he then assumed that the same concepts were adequate when the processes occurred simultaneously, that the incremental work was the sum of the separate expressions, and that interaction could be sufficiently taken into account by merely inserting product terms in the thermodynamic potential. This procedure now appears to have been incorrect. The general form of the relevant equations suggests that in many cases the differences affect only second-order terms in the final formulas and are therefore not significant over the range to which Voigt's theory applies. Nevertheless, the difference of interpretation is important in principle, and there may be cases in which the discrepancies are significant.

(b) For an isotropic ferromagnetic material undergoing small, approximately reversible changes from an initial state, or for such a material in general if hysteresis is neglected, F may be expressed as a quadratic function of the strains, in which the coefficients of the terms of zeroth, first, and second degree are in general all func-

[15] Woldemar Voigt, *Lehrbuch der Kristallphysik* (B. G. Teubner, Leipzig, 1910 and 1928); Walter Guyton Cady, *Piezoelectricity* (McGraw-Hill Book Company, Inc., New York, 1946), first edition. Voigt's sign convention for the stresses is opposite to Love's, which has been adopted here; Cady adheres to Voigt's convention.

tions of the specific magnetization components M_x, M_y, M_z. If hysteresis is negligible or if the magnetization direction has remained constant since the last demagnetization, symmetry about the magnetization direction may be used to deduce relations among the coefficients. A number of relations, e.g., between the direct and converse magnetostrictive phenomena, may be derived by this method.

It is legitimate to define X_x^*, etc., as "stresses," provided the differential equations of equilibrium and the boundary conditions are expressed correctly in terms of the stresses as so defined. The non-uniqueness of the "stresses" is illustrated here again: if the **B** equations are used, Eqs. (4.4–1)–(4.4–4) are replaced by

$$X_x^{*\prime} \equiv X_x - \tfrac{1}{2}\gamma(M_y^2 + M_z^2) = X_x^* - \tfrac{1}{2}\gamma M^2, \cdots, \quad (4.4\text{–}1')$$

$$X_y^{*\prime} \equiv Y_x^{*\prime} \equiv \tfrac{1}{2}(X_y + Y_z) = X_y^*, \quad (4.4\text{–}2')$$

$$dF' = B_x dM_x + \cdots + V\{X_x^{*\prime} de_{xx} + \cdots + X_y^{*\prime} de_{xy} + \cdots\}, \quad (4.4\text{–}3')$$

$$B_x = \partial F'/\partial M_x, \cdots, VX_x^{*\prime} = \partial F'/\partial e_{xx}, \cdots, VY_z^{*\prime} = \partial F'/\partial e_{yz}, \cdots. \quad (4.4\text{–}4')$$

Whatever "stresses" are used, it is important to use the form of the equations of motion and boundary conditions that holds for that definition of the stresses. The ambiguity of the stress concept and the importance of using mutually consistent forms of the stress-strain-field polarization relations, of the equations of motion, and of the boundary conditions have gone unnoticed in most treatments of these relations: even the thorough and systematic Voigt is open to criticism in this respect. Some of the differences of opinion in regard to conclusions from energy arguments may have resulted from failure to recognize that more than one definition of stress is possible.

The "stresses" $X_x^*, X_x^{*\prime}$, which are derivatives of thermodynamic potentials with respect to strains, must not be confused with the "stresses" $\bar{X}_x, \bar{X}_x'$ of Eqs. (1.5–4)–(1.5–8), which originated upon introduction of fictitious charge and current densities and (it now appears) have only formal significance.

4.5 Epilogue

It was not to be expected that anything essentially new in macroscopic electrostatics and magnetostatics could be discovered in 1950. Nevertheless, this attempt to derive the old results by a more direct method has revealed several interesting facts.

One is that all the general formulas can be derived from the basic law for the force and couple on a dipole in an external field and from elementary mechanical principles. It is not necessary to introduce supplementary hypotheses in regard to the localization of energy or the nature of the "effective field" at points inside a polarized medium. All the results given here have been derived without attributing to **E**, **D**, **H**, and **B** any significance other than such as may follow directly from their definitions as quantities computed by a specified integration process. All important results, moreover, have been derived in two equivalent forms, one of which corresponds formally to the interpretation of **H** as the effective field, and the other to a similar interpretation of **B**. By linear combination of these two forms, others may be obtained that correspond to the view that the effective field is $\alpha\mathbf{H} + (1-\alpha)\mathbf{B}$, where α is an arbitrary constant between 0 and 1. It is clear that the choice of **H** or of **B** as independent variable in macroscopic magnetostatics is purely a matter of convenience and has nothing to do with one or the other's being more "fundamental."

Another interesting fact is that "stresses" need to be defined with more care than has been generally recognized. As long as only forces and torques are to be computed, there is considerable arbitrariness in the definition, because certain terms can be expressed either as volume or as surface integrals; thus, the "stresses" depend on whether the "magnetic" forces are computed by use of dipole densities, of pole densities, or of amperian current densities. When properties of matter are to be established by use of thermodynamic potentials, the arbitrariness is more limited; but it is still possible to separate the force into a "long-range" term and a "short-range" or stress term in more than one way, because the long-range term—however calculated—always includes a contribution from short distances, and therefore the short-range term has no unique definition. The present analysis has produced, altogether, five different definitions of the stresses, each of which seems natural

and inevitable when the subject is approached from some particular point of view.

A third conclusion is that the introduction of narrowly restrictive assumptions in regard to the properties of the matter considered is not necessary in order to carry the calculation through to a useful conclusion. The standard treatments are almost all limited to linear, isotropic materials—despite the facts that ferromagnetic materials are not linear, that solid materials are not usually isotropic, and that it is very difficult to avoid contradictions when a solid material first is assumed to be magnetically linear and then is assumed to have magnetostrictive properties. There seems to be no reason for this procedure except tradition; the retention of completely general properties, up to the point where the formulas are ready to be specialized to particular classes of material, has led in the present treatment to no enormous complexities.

Because the aim here was to avoid basing the general conclusions (as distinguished from the specific properties of particular classes of matter) on energy arguments, no reference has so far been made to treatments based on such methods, except in connection with the properties of fluids. The classical treatment by such methods, as presented by Abraham-Becker (see reference 5, Part I) and by Stratton (see reference 8, Part I), makes considerable use of the linearity and isotropy assumptions, which limit the generality of the results obtained. More general analyses of energy relations have been undertaken by Guggenheim,[16] Livens,[17] and Stoner;[18] references to earlier discussions by Larmor, Livens, and others will be found in these papers. Few of the formulas derived by these authors are directly comparable with ones derived here. The discussions of Livens and of Guggenheim (who express divergent views on a number of questions) concern partly very general relations and partly the behavior of materials with rather special magnetic properties; the work of Stoner is aimed primarily at obtaining thermodynamic relations between magnetic quantities and such quantities as specific heat.

More recently, Smith-White[19] has derived formulas which he considers incompatible with the generally accepted ones for the force on a portion of a polarized fluid. His calculation is open to the objection that his basic formula (2) attributes a particular significance to the field vector $\mathbf{E}$. Nevertheless, his results are apparently correct if correctly interpreted. His own equations (2) and (3) evaluate a term $\mathbf{F}$ corresponding to $M\nabla H$ in Eq. (2.1–3) of the present article; they are correct if used in conjunction with a "pressure" p' (Sec. 2.1). His equation (1), quoted from Helmholtz, corresponds to a different analysis of the force into a "long-range" term $\mathbf{F}^{(h)}$ and a "stress" term; it is correct if used in conjunction with a "pressure" $p(\rho)$ (Sec. 2.3). Smith-White's equation (4) for the difference $\mathbf{F}^{(h)}-\mathbf{F}$ can be derived directly from $\mathbf{F}^{(h)}-\nabla p(\rho)=\mathbf{F}-\nabla p'$ and from Eqs. (2.3–5) and (2.3–6). There is, therefore, no discrepancy, but only a confusion between different analyses of the total force into long-range and short-range terms.

Since Smith-White followed Livens's method, it is likely that there is a similar explanation of the discrepancies that Livens thought he had detected between his own results and Guggenheim's. The calculations of Sec. 2.2 confirm Guggenheim's results in cases in which a direct comparison is possible.

The following recent papers all contain calculations based on the maxwell stress tensor, i.e., essentially on Eq. (2.4–6′). Diesselhorst[20] discusses topics covered in Sec. 2.2 of the Report and arrives at similar conclusions. He derives Eq. (2.5–9), given above, by a method not quite as simple as that of Sec. 2.5, but much simpler than that of Page and Preston (references 10 and 11, Part I) and that of Sommerfeld and Ramberg[21]—all of whom solve the potential problem directly instead of reducing it to one already

[16] E. A. Guggenheim, Proc. Roy. Soc. (London) **A155**, 49–70 and 70–101 (1936); Phil. Mag. **33**, 479 (1942); Phys. Rev. **68**, 273–6 (1945).

[17] G. H. Livens, Phil. Mag. **36**, 1 (1945) and **38**, 453–479 (1947); Phys. Rev. **71**, 58–63 (1947); Proc. Cambridge Phil. Soc. **44**, 534–545 (1948).

[18] Edmund C. Stoner, Phil. Mag. **19**, 565–588 (1935) and **23**, 833–857 (1937).

[19] W. B. Smith-White, Phil. Mag. **40**, 466–479 (1949); see also Nature **166**, 689–691 (1950).

[20] H. Diesselhorst, Ann. Physik **3**, 11–30 (1948). Unfortunately, I did not become aware of this paper until after publication of the Report; it should have been cited in Appendix B of the Report, especially in connection with Eq. (B-9).

[21] A. Sommerfeld and E. Ramberg, Ann. Physik **8**, 46–54 (1950).

solved. Why the last-named authors did so is not clear, since they cite Diesselhorst's paper. Döring[22] is concerned with the choice between the definitions $\mathbf{m}_p$ and $\mathbf{m}_I$ (Sec. 2.5) of the magnetic moment of a magnet, and between $\mathbf{H}$ and $\mathbf{B}$ as the more fundamental field vector. The hypothesis underlying the present investigation, and verified by its results, is that no such choice is necessary for the calculation of physically meaningful quantities; that to every $\mathbf{H}$-formula there corresponds an equally valid $\mathbf{B}$-formula and vice versa; and that the use of one or the other formula in any particular problem is purely a question of convenience. Finally, Sommerfeld and Bopp[23] attempt to derive a force formula by postulating that the force density is the divergence of a symmetrical tensor. They consider two possible forms for this tensor, namely, a generalization of Eq. (2.4–6′) (the maxwell stress tensor) and a "modified stress tensor" (attributed to Kneissler) that gives, for a magnet in a Class A fluid, a torque $(\kappa_m)_A$ times as large as does the maxwell tensor. Now the derivation of Eq. (2.4–6′) given here requires no postulates but follows directly from the law of force between dipoles, the analysis of stress, and elementary mechanics; everything else, including the introduction of field vectors, is mere mathematical manipulation. Therefore, the "modified stress tensor" must be rejected. The generalization for an arbitrary material also requires no postulates. The fourth term in Eq. (1.5–6) is the x-component of $(-\nabla\cdot\mathbf{M})\mathbf{H}=(1/\gamma)(\nabla\cdot\mathbf{H})\mathbf{H}$; the fourth term in Eq. (1.5–6′) is the x-component of $(\nabla\times\mathbf{M})\times\mathbf{B}=(1/\gamma)(\nabla\times\mathbf{B})\times\mathbf{B}$. The volume integral of either of these expressions can be transformed to a surface integral by use of Eq. (1.3–1) or (1.3–1′) and of $\nabla\times\mathbf{H}=\nabla\cdot\mathbf{B}=0$; the transformation of the first three terms to surface integral form is elementary. Thus, the combined elastic and magnetic force can be interpreted as the effect of a "stress" of the maxwell type and separated into "elastic" and "magnetic" parts in various ways, as convenience may dictate in the particular problem.[24] It is futile to debate whether one or another expression for the magnetic force density is "correct": each of various ones is correct if used with the appropriate "elastic" stresses, and meaningless otherwise. It is likewise futile to attempt to analyze "magnetic" forces in a deformable body without considering its elastic and magnetostrictive properties.

One reason for the paucity of generally valid formulas, in treatments based on energy considerations, is that the energy method leads most naturally to a volume integral involving rates of change or variations at fixed points, whereas any formulas describing properties of matter must involve mass integrals and rates of change or variations with respect to axes attached to matter. The authors cited have employed variations of the former type and, therefore, in order to obtain usable results, have been forced to limit their discussions to fluids, rigid bodies, or other special cases. A calculation based on the energy approach and on a variational principle can be carried out without any such limitations, if variations of the other type are used; it leads to equations of motion and boundary conditions equivalent to those derived here. The details of such a calculation will not be given here, since it represents an essentially different method of attack on the problem.

The direct calculation of forces has turned out to be fairly simple. In some cases, such as fluid electrostriction, it has led without great difficulty to formulas of which the conventional derivation, by energy methods, is complicated and not entirely convincing. In the more intricate case of the elastic solid, the results suggest that some steps in the traditional treatments require careful scrutiny.

APPENDIX A

Derivation of Eq. (4.2–9)

Let U_{12} represent the right member of Eq. (4.2–8) before going to the limit $S_1, S_2 \to S$. Then, by setting $\mathbf{H}_{12}$

[22] W. Döring, Ann. Physik **6**, 69–88 (1949).

[23] A. Sommerfeld and F. Bopp, Ann. Physik **8**, 41–45 (1950).

[24] *Note added in proof:*—For another proof of this theorem, see Joseph Slepian, Proc. Natl. Acad. Sci. U. S. **36**, 485–497 (1950). Slepian begins with definitions of the "electric" (long-range) and "mechanical" (short-range) forces equivalent to those of Sec. 1.1. He shows that the mechanical force so defined is not derivable from a tensor. He resolves this "dilemma" by changing the definitions, but the resulting separation of the total force tensor into electrical and mechanical parts is not unique. In my theory there is no postulate about the derivability of forces from tensors; there is therefore no dilemma, and the original definitions can be retained.

$=-\nabla_2 V_{12}$ and transforming in the usual manner, it can be shown that

$$U_{12}\equiv -\frac{1}{2}\int_{\tau_2}\mathbf{M}_2\cdot\mathbf{H}_{12}d\tau$$
$$=\frac{1}{2}\int_{\tau_2}\rho_2 V_{12}d\tau_2+\frac{1}{2}\int_{S_2}\sigma_2 V_{12}dS_2, \quad \text{(A-1)}$$

where $\rho_2=-\nabla_2\cdot\mathbf{M}_2$, $\sigma_2=\mathbf{n}_2\cdot\mathbf{M}_2$. If V_{12} is written explicitly in terms of ρ_1 and σ_1, it is seen that U_{12} is symmetrical in the subscripts 1 and 2. Reversal of the steps with subscripts interchanged is permissible; for although τ_1 is divided into two parts by S_2, there is no discontinuity of V_{21} or of $\mathbf{M}_1$ across this surface, and, therefore, it contributes no surface integral. Thus,

$$U_{12}=-\frac{1}{2}\int_{\tau_2}\mathbf{M}_2\cdot\mathbf{H}_{12}d\tau_2=-\frac{1}{2}\int_{\tau_1}\mathbf{M}_1\cdot\mathbf{H}_{21}d\tau_1, \quad \text{(A-2)}$$

and

$$\frac{dU_{12}}{dt}=\frac{d_1U_{12}}{dt}+\frac{d_2U_{12}}{dt}$$
$$=-\frac{1}{2}\frac{d_1}{dt}\int_{\tau_1}\mathbf{M}_1\cdot\mathbf{H}_{21}d\tau_1-\frac{1}{2}\frac{d_2}{dt}\int_{\tau_2}\mathbf{M}_2\cdot\mathbf{H}_{12}d\tau_2, \quad \text{(A-3)}$$

where the differentiation d_1/dt is carried out by freezing S_2 and $\mathbf{M}_2$ while S_1 and $\mathbf{M}_1$ vary, and vice versa.

The first term is the limit, as $\Delta t\to 0$, of

$$-\frac{1}{2\Delta t}\Big\{\int_{\tau_1+\Delta\tau_1}(\mathbf{M}_1+\Delta\mathbf{M}_1)\cdot\mathbf{H}_{21}d\tau_1$$
$$-\int_{\tau_1}\mathbf{M}_1\cdot\mathbf{H}_{21}d\tau_1;\Big\} \quad \text{(A-4)}$$

or (by omission of second-order infinitesimals) of

$$-\frac{1}{2\Delta t}\Big\{\int_{\Delta\tau_1}\mathbf{M}_1\cdot\mathbf{H}_{21}d\tau_1+\Delta t\int_{\tau_1}\frac{\partial\mathbf{M}_1}{\partial t}\cdot\mathbf{H}_{21}d\tau_1\Big\}. \quad \text{(A-5)}$$

The volume $\Delta\tau_1$ is the volume between the original surface $S_1(t)$ and the surface $S_1(t+\Delta t)$ bounding the same matter after a time interval Δt, during which each point of S_1 has experienced a vector displacement $\mathbf{v}_1\Delta t$; therefore, $d\tau_1$ may be replaced by $\mathbf{n}_1\cdot\mathbf{v}_1 dS_1\Delta t$. The limit $\Delta t\to 0$ gives

$$-\frac{1}{2}\frac{d_1}{dt}\int_{\tau_1}\mathbf{M}_1\cdot\mathbf{H}_{21}d\tau_1=-\frac{1}{2}\int_{1S}\mathbf{M}_1\cdot\mathbf{H}_{21}\mathbf{n}_1\cdot\mathbf{v}_1 dS_1$$
$$-\frac{1}{2}\int_{\tau_1}\frac{\partial\mathbf{M}_1}{\partial t}\cdot\mathbf{H}_{21}d\tau_1, \quad \text{(A-6)}$$

and a similar formula holds for the second term in Eq. (A-3). The limit $S_1, S_2\to S$ now gives

$$\frac{dU_m}{dt}=-\frac{1}{2}\int_S\mathbf{M}\cdot(\mathbf{H}_1^{+}+\mathbf{H}_1^{-})\mathbf{n}\cdot\mathbf{v}dS-\int_\tau\frac{\partial\mathbf{M}}{\partial t}\cdot\mathbf{H}_1 d\tau$$
$$=-\frac{1}{2}\int_S\mathbf{M}\cdot(\mathbf{H}_1^{+}+\mathbf{H}_1^{-})\mathbf{n}\cdot\mathbf{v}dS$$
$$-\int_\tau\mathbf{H}_1\cdot\Big[\frac{D\mathbf{M}}{dt}+\boldsymbol{\Omega}\times\mathbf{M}-\mathbf{v}\cdot\nabla\mathbf{M}\Big]d\tau. \quad \text{(A-7)}$$

Now

$$\int_\tau\mathbf{H}_1\cdot(\mathbf{v}\cdot\nabla\mathbf{M})d\tau=\int_\tau[H_{1x}\mathbf{v}\cdot\nabla M_x+\cdots]d\tau$$
$$=\int_S\mathbf{n}\cdot\mathbf{v}(H_{1x}M_x+\cdots)dS-\int_\tau[M_x\nabla\cdot(\mathbf{v}H_{1x})+\cdots]d\tau$$
$$=\int_S\mathbf{M}\cdot\mathbf{H}_1\mathbf{n}\cdot\mathbf{v}dS-\int_\tau[\mathbf{M}\cdot(\mathbf{v}\cdot\nabla\mathbf{H}_1)+\mathbf{M}\cdot\mathbf{H}_1\nabla\cdot\mathbf{v}]d\tau$$
$$=\int_S\mathbf{M}\cdot\mathbf{H}_1\mathbf{n}\cdot\mathbf{v}dS-\int_\tau[\mathbf{v}\cdot(\mathbf{M}\cdot\nabla\mathbf{H}_1)$$
$$+\mathbf{M}\cdot\mathbf{H}_1\nabla\cdot\mathbf{v}]d\tau, \quad \text{(A-8)}$$

since $\nabla\times\mathbf{H}_1=0$. Therefore,

$$\frac{dU_m}{dt}=-\frac{1}{2}\int_S\mathbf{M}\cdot(\mathbf{H}_1^{+}-\mathbf{H}_1^{-})\mathbf{n}\cdot\mathbf{v}dS-\int_\tau\Big[\mathbf{v}\cdot(\mathbf{M}\cdot\nabla\mathbf{H}_1)$$
$$+\boldsymbol{\Omega}\cdot\mathbf{M}\times\mathbf{H}_1+\mathbf{H}_1\cdot\Big(\frac{D\mathbf{M}}{dt}+\mathbf{M}\nabla\cdot\mathbf{v}\Big)\Big]d\tau. \quad \text{(A-9)}$$

But

$$\mathbf{M}\cdot(\mathbf{H}_1^{+}-\mathbf{H}_1^{-})=\mathbf{M}\cdot[\gamma M_n\mathbf{n}]=\gamma M_n^2 \quad \text{(A-10)}$$

and

$$\Big(\frac{D\mathbf{M}}{dt}+\mathbf{M}\nabla\cdot\mathbf{v}\Big)d\tau=\Big[\frac{D(\rho\mathbf{M})}{dt}-\mathbf{M}\frac{D\rho}{dt}\Big]d\tau$$
$$=\frac{D\mathbf{M}}{dt}\rho d\tau=\frac{D\mathbf{M}}{dt}dm. \quad \text{(A-11)}$$

Substitution of Eqs. (A-10) and (A-11) in Eq. (A-9) gives Eq. (4.2–9).

Fulbright Awards for 1952–1953

Fulbright awards for the year 1952–53 offer opportunities for university lecturing and advanced research in education, the humanities, the natural sciences, and the social sciences. Countries in which positions may be available are The United Kingdom and Colonial Dependencies, Norway, Netherlands, Belgium and Luxembourg, France, Austria, Italy, Greece, Egypt, Turkey, and Iran. It is expected that approximately 230 awards will be made.

Applications can be accepted until about the middle of October. The latest acceptable mailing date is October 15, 1951. Application forms and additional information may be obtained from the Executive Secretary, Committee on International Exchange of Persons, Conference Board of Associated Research Councils, 2101 Constitution Avenue, Washington, D. C.

A Unified Approach to Physics

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The general elementary course is unified on a basis of underlying theory. Part I introduces seriatim five fundamental concepts to serve a dimensional analysis for the whole of physics. Selected topics make the nature of these concepts clear, e.g., (1) geometric optics for *length*, (2) kinematics for *time*, (3) statics of a rigid body for *force*, (4) electrolysis for *electricity*, and (5) thermometry and calorimetry for *temperature*. Part II contains five sections which cut horizontally across the conventional subdivisions of physics. (1) *Energetics*, in which dynamics, thermodynamics, and the relativity mass-energy relation find a place; (2) *Flow Phenomena* stress the parallelisms between fluids, heat, and electricity; (3) *Field Phenomena* apply the inverse square law to gravitation, electro- and magnetostatics, photometry, and radiation; (4) *Periodic Phenomena*, starting with circular and simple harmonic motions, end with waves of all kinds; and (5) *Quantum Phenomena* deal with the elemental particles and the indeterminancy principle.

WE repeat an appeal made over twenty-five years ago[1] for an approach to the teaching of physics in which the conventional compartments: mechanics, sound, heat, light, electricity, and magnetism, followed by modern physics, are abandoned in favor of an organization based upon underlying theory; and we support that appeal by a report on a quarter of a century of use of such an approach in our first-year course in physics.

Physics has long since ceased to be merely a descriptive science. Should physics teachers continue to use the approach of a hundred years ago as if they still dealt with the world of nature directly with their hands and ears and eyes? To be sure, physics is an experimental science. Following the precept of Professor Bridgman, "The concept is synonymous with the corresponding set of operations," what we *mean* is what we *do*. But, when modern instrumentation gives us a black box with several decades of switches on the top, will our senses tell us at once whether it contains a resistance, a capacitance, or an inductance? Even when we take the box apart and obtain a piece of wire from its interior and examine it in the electrical measurements laboratory, there still remains the question, is it *L*, *C*, or *R*? Nor must we forget that our nearest approach to physical reality is in those direct contacts with nature which constitute our primary sensations. "If we see, hear, or touch something, it is clearly a given fact which no skeptic can endanger," said Max Planck. Nevertheless, physics is more than the manipulation of dial switches or the recording of our own bodily sensations. The physicist of today is no longer a baby on the lawn playing with his toys, who reaches for the teddy bear and the white fluffy cloud overhead with the same hand. It's time to say, "Triangulate, my child, triangulate."

The philosophy behind our unified approach is to present advanced theories from an elementary point of view. Such an aim does not result in a mere popularization, which might picture the apparent paradoxes of relativity on a comic strip and fail to derive the essential equations. Nor do we employ that historical treatment which talks *about* physics and tells of the trials and tribulations of the masters. This latter approach may give the strategy of physics, but not its tactics. For tactics, in the sense that the word was used by Alfred Mahan, takes over when hostile fleets or armies have made contact. Our student experiences his science first hand. To him physics is to be *physics*.

One gains an advantage in using an approach based on the underlying theory rather than the outward aspect. Organization in terms of theory makes for economy of effort. Herein lies a solution for the problem of the ever-increasing content of physics. Consider, for example, the unifying effect of the two great conservation principles, those of mass and energy, as they were

[1] N. C. Little, Science 62, 286 (1925).

developed during the nineteenth century. Although the turn of the century brought the isolation of the electron, the discovery of radioactivity, and finally, the splitting of the atom, we are left with but half a dozen different fundamental particles in place of a hundred different atoms, and the two laws of conservation have become fused into one. The content of basic theory, therefore, tends to contract with time. An approach founded on theory need not include the latest invention nor describe the most recent scientific marvel. If radar enters, it is not on account of its importance in World War II, but because Maxwell beckons. Nevertheless, we present the science of physics as a whole. There are no gaps. Any particular phenomena or special instrument not discussed in detail can find a niche waiting for it. The framework of the house of physics is made firm and fixed. It may be necessary to "do over" a room now and then, or bring home a new piece of furniture, but we hold to a single architectural plan based on the broad lines of underlying theory.

Lord Kelvin's remark, "When you can measure what you are talking about and express it in numbers, you know something about it, but when you cannot, your knowledge is of a meager and unsatisfactory kind; it may be the beginning of knowledge, but you have scarcely in your thoughts advanced to the stage of a science," is almost a cliché. Yet before one can measure, there must first be set up a consistent system of defining equations, in which certain concepts are selected as primary or fundamental and all other secondary or derived. How their choice shall be made is, of course, quite arbitrary. The traditional choice is three fundamental concepts, mass, length, and time. These concepts, however, are often forced to play a role to which they are not particularly well suited. Acting their part reasonably well in mechanics, they become awkward and self-conscious in other parts of the drama of physics. In the scenes on electricity and magnetism they have the bad habit of appearing with fractional exponents and, what is worse, these exponents depend upon whether the derived concept enters upon the stage from the door marked electricity or whether it appears from the magnetism entrance. For example, electric current may have the dimensions $[M^{\frac{1}{2}}L^{\frac{1}{2}}T^{-2}]$ or $[M^{\frac{1}{2}}L^{\frac{1}{2}}T^{-1}]$. The selection of five fundamental concepts, length, time, force, quantity of electricity, and temperature leads to a definitive dimensional analysis which may be used throughout the whole of physics. On account of its unifying effect, this choice is made in our approach. As the following brief outline shows, our plan in the first part of the course, is to introduce these five fundamental concepts, seriatim, as the building blocks of the structure of physics. With the introduction of each, some limited section of physics is treated as illustrative of each concept's nature and use. The second part of the course, in dealing with types of phenomena which have the same underlying theory, cuts horizontally across the traditional subdivisions of mechanics, sound, light, etc. The division of the student's time between the two parts is in the ratio of three to five.

PART I. FIVE FUNDAMENTAL CONCEPTS

1. Introducing Length. Angles, Areas, and Volumes; Specific Gravities and Densities; Geometrical Optics.

2. Time, Motion with One Degree of Freedom. Kinematics of Translation; Kinematics of Rotation.

3. Force. Concurrent Forces; Statics of Rigid Bodies.

4. Quantity of Electricity. Electrical Nature of Matter; Electrolysis; Properties of Neutral Matter.

5. Temperature. Thermometry; Calorimetry; Change of State.

PART II. FIVE TYPES OF PHENOMENA

1. Energetics. Work, Power, Kinetic Energy; First Law of Thermodynamics; Dynamics of Translation and Rotation; Second Law of Thermodynamics; Relativity, Mass-Energy Relation.

2. Flow Phenomena. Factors Controlling Flow of Liquids; Heat and Electricity; Dc Circuit Theory; Ideal Flow,—Bernoulli Principle, Thermoelectric Effects; Quasi-Flow,—Electric and Magnetic Flux; Transient Flow of Electricity and Ac Theory.

3. Field Phenomena. Local and Universal Gravitational Fields; Electro- and Magnetostatics; Radiation; Electrodynamics, Ampère's Law.

4. Periodic Phenomena. Centripetal Acceleration and Gyroscopic Precession; Simple Harmonic Motion; Wave Phenomena Applied to All Types of Reflection, Refraction, Interference; Diffraction, Polarization.

5. Quantum Phenomena. Electron, Mass, and Specific Charge; Bohr-Rutherford Atom and Periodic Table; Kinetic Theory and Gaseous Conduction; Photon and Wave Picture of Matter; Nuclear Physics.

A brief outline requires some elucidation even to the experienced teacher. He will be interested in more detailed information of our approach to the concept of mass. The student is first introduced to this concept in the section introducing length, in order to bring out the difference between an abstract concept of the mathematician and a concrete one of the physicist. A preliminary discussion of the distinction between mass and weight is given, with attendant definitions of density and specific gravity. This incomplete treatment of mass is a digression from a purely logical scheme of development; but it enables the student to apply his geometry to real objects, and, if the character of the digression is clearly pointed out, it serves to emphasize the logic behind the precise definition of mass when it finally is given in the section on energetics. The student again meets mass in the treatment of electrolysis and calorimetry before he is in a position to analyze it dimensionally. These preliminary encounters with the concept, however, serve to impress upon him the need of a more rigorous definition and to motivate the eventual complete treatment under dynamics.

Another break in the strictly consistent dimensional development occurs in the treatment of calorimetry. Heat is introduced before energy has been defined. The student is told, however, of the unsatisfactory nature of a unit defined in terms of a particular substance like water, and is promised a dimensional analysis shortly.

Some teachers will be surprised at the introduction of geometrical optics so early in the course, but in view of the optical aids to measurement which the student must encounter in the laboratory, there is little reason to postpone the theory of their operation until the story of light is completed under wave and quantum phenomena.

With quantity of electricity as a fundamental concept, it is not necessary to confront the student immediately with the force relations given by Coulomb's and Ampère's laws. Introduced as associated with mass particles in a simple picture of the atom and with an accompanying treatment of electrolysis, electricity appears again under flow phenomena. An opportunity is afforded to distinguish between irreversible flow with attendant I^2R conversion of electrical energy into heat and reversible flow with the linear law associated with Peltier heat. Electric and magnetic flux are introduced as theoretical aids associated with a quasi-flow (time factor omitted) in the simple geometry of the parallel-plate capacitor and the toroidal coil. A natural basis for the concepts of inductance and capacitance is thus laid. These latter concepts (with the time factor readmitted) lead to the concept of reactance and a simple treatment of alternating currents as flow phenomena. Finally, under field phenomena, after the student is somewhat familiar with electricity and magnetism, the basic force relations are given.

Pedagogical advantages resulting from this unified approach are well illustrated in the treatment of kinematics. A change due to difference in direction, but not in magnitude, is difficult for the student to understand. The traditional organization considers centripetal acceleration and gyroscopic precession early in the course. A sampling of recently published textbooks shows that the treatment of both is completed in the first 16 percent of their pages. Not only does our approach postpone their treatment to the last quarter of the course, but it is also able to stress the essential similarity of the vector theory of each.

Parallelisms of the sort just mentioned abound to make our approach truly unifying. A great deal of economy in exposition results in organizing elementary physics on the basis of the underlying theory. The student obtains a connected view of the science as a whole, and the emphasis is placed where it should be in a science which is fundamental to all others.

Atwood's Machine and the Teaching of Newton's Second Law

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(Received January 12, 1951)

The use of Atwood's machine in illustrating the principles of the application of Newton's second law to a simple physical system, whose connected parts have both rotational and translational motion, is shown. A diagram which shows the forces acting on all the component masses is drawn, and this picture facilitates the setting-up of the appropriate $F=ma$ and $\tau=I\alpha$ equations. The moment of inertia of the pulley, the friction in its bearings, and the mass of the string are included in the accelerating system in a natural way. The observed acceleration of the hanging masses can be compared with that computed by this method.

AN experiment which can be used to determine the acceleration of gravity or to provide a check on the validity of Newton's second law was devised by George Atwood in 1784.[1] Elementary laboratory experiments using essentially the same equipment are still quite popular today. Equal masses are hung over a pulley by a light cord, and an unbalanced weight in the form of a rider attached to one of the masses causes the system to accelerate. During the motion the rider is removed by a fixed stop; and now the system moves with a constant velocity, which can be found by measuring the time for one of the weights to travel a given distance. The time during which the acceleration takes place is also measured, and the actual average acceleration is then the ratio of the velocity acquired divided by this time. This procedure is described in several well-known physics laboratory textbooks.[2] (Of course, the acceleration can also be found from the familiar $s=\frac{1}{2}at^2$, but this viewpoint does not as clearly show to the student experimenter the succinct relationship between acceleration and velocity.) The observed acceleration may be compared with that expected on the basis of Newton's second law; or, the second law can be accepted as valid, and, by the application of it to this simple physical situation, the ratio of the weight of a body to its mass, that is, g, can be found.

Undoubtedly, reasonably good experimental results can be obtained in the laboratory, even if the effects of friction in the pulley, moment of inertia of the pulley, and weight of the string are neglected. To improve the accuracy, the first of these corrections can be found by an extrapolation method,[3] and the second is also taken into account by a rather simple physical argument, as Miller[1] shows. The effect of the weight of the cord (but not its mass!) can be eliminated by using a continuous string, which should be very flexible. Too often, however, we tend to introduce corrections such as these *ad hoc;* it is the purpose of this note to discuss the equations governing Atwood's machine from a more unified viewpoint.

In our sophomore physics course, the Atwood machine is one of several devices used to illustrate the application of the often-misunderstood equation, $F=ma$, to actual physical problems. We stress the technique of isolating the several masses in a physical system and drawing a force diagram for each. From these diagrams there are written down the equations embodying the principles that the net unbalanced force is equal to the mass of the body times the acceleration of the center of mass, and that the torque about the center of mass is equal to the moment of inertia of the body about its center of mass times the angular acceleration that it experiences. These

[1] George Atwood, *A Treatise on the Rectilinear Motion and Rotation of Bodies; with a Description of Original Experiments Relative to the Subject* (Cambridge, 1784).

[2] For example, Millikan, Roller, and Watson, *Mechanics, Molecular Physics, Heat, and Sound* (Ginn and Company, New York, 1937); J. Duncan and S. G. Starling, *A Textbook of Physics* (Macmillan Company, New York, 1918); and D. C. Miller, *Laboratory Physics* (Ginn and Company, New York, 1932). Miller's analysis takes into account the effect of the rotational inertia of the pulley.

[3] K. H. Fried and W. H. Mais, Am. J. Phys. **12**, 210 (1944). A fixed total accelerated mass is used, and the accelerations observed are plotted against the mass *differences* (obtained by transferring weights from one side to the other) which cause them. The weight difference extrapolated to zero acceleration is the friction force.

equations can often be added together (after some slight modification of the torque equations), eliminating the internal tensions and compressions and giving a single expression for the acceleration as a function of the other parameters. At worst, one gets n linear equations in n unknowns. Somehow, there is always considerable student resistance to this point of view; it is "easier" just to say that the net force is equal to the total mass times its acceleration; besides, it takes some practice to get any facility in doing problems this way. Unfortunately, however, there are a large number of problems that do not succumb to the "easy" attack. We therefore treat the Atwood machine as an exercise on Newton's second law.

Consider, then, an Atwood machine consisting of a fixed pulley of moment of inertia I, radius r, and weight W_p, over which are hung masses M_1 and M_2 by means of a continuous tape, the sections of which are labeled $m_1, m_2, \cdots, m_6$ (shown in force diagram, Fig. 1*). The force R is the reaction exerted on the pulley by the bearings. It is necessary to assume that the frictional torque in the bearings, τ_{fr}, remains constant during the whole experiment, that air resistance may be neglected, and that the tape is perfectly flexible, does not slip on the pulley, and hangs in a semicircle at its bottom.[4] When M_1 and M_2 are adjusted so that the system does not accelerate but moves with constant speed when started, the following equations result from application of Newton's second law:

$$T_2+m_2g-T_1=0, \tag{1}$$

$$T_3+M_1g-T_2=0, \tag{2}$$

$$T_4+m_3g-T_3=0, \tag{3}$$

$$T_5r'-T_4r'=0, \tag{4}$$

$$T_5+T_4-m_4g=0, \tag{5}$$

$$T_6-T_5-m_5g=0, \tag{6}$$

$$T_7-T_6-M_2g=0, \tag{7}$$

$$T_8-T_7-m_6g=0, \tag{8}$$

$$T_1r-T_8r-\tau_{fr}=0, \tag{9}$$

$$m_1g+W_p+T_1+T_8-R=0. \tag{10}$$

Adding together all these equations except Eqs. (5) and (10), after dividing Eq. (4) by r' and Eq. (9) by r, we get

$$\{(m_2+m_3)-(m_6+m_5)+(M_1-M_2)\}g-\tau_{fr}/r=0. \tag{11}$$

The weight of the parts of the tape cancels out, and we are left with the familiar result that the difference in weight times its lever arm is sufficient to overcome the torque due to friction.

Now a rider of mass M_3 is added. Application of the equations $F=ma$ and $\tau=I\alpha$ gives for the accelerated case

$$T_2+m_2g-T_1=m_2a, \tag{12}$$

$$T_3+(M_1+M_3)g-T_2=(M_1+M_3)a, \tag{13}$$

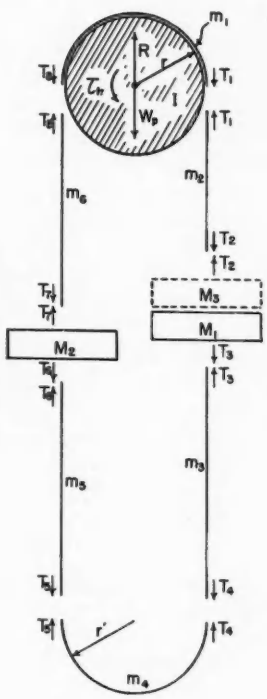

FIG. 1. Force diagram for Atwood's machine. For the sake of simplicity, the vectors showing the weights of the bodies are omitted. These may be considered to act at the corresponding centers of gravity.

* The author wishes to express his thanks to Mr. Raymond G. Olsson, who prepared Fig. 1.

[4] The problem of determining the shape of a string which cannot support a shear, when it is being accelerated as in the Atwood machine, is an extremely difficulty one. The forces acting on each element of length are the weight of the element and the tensions applied by the two neighboring elements, and these forces produce both the translational acceleration of the cord and its centripetal acceleration. Two extremely complicated second-order differential equations, involving the x- and y-location of the element and the tension in the cord at that point, can be found; but these equations apparently cannot be solved in closed form.

$$T_4+m_3g-T_3=m_3a, \tag{14}$$

$$T_5r'-T_4r'=m_4r'^2\alpha=m_4r'^2(a/r'), \tag{15}$$

$$T_5+T_4-m_4g=0, \tag{16}$$

$$T_6-m_5g-T_5=m_5a, \tag{17}$$

$$T_7-M_2g-T_6=M_2a, \tag{18}$$

$$T_8-m_6g-T_7=m_6a, \tag{19}$$

$$T_1r-T_8r-\tau_{fr}=(I+m_1r^2)\alpha =(I+m_1r^2)(a/r), \tag{20}$$

$$m_1g+W_p+T_1+T_8-R=0. \tag{21}$$

When all except Eqs. (16) and (21) are added, after dividing Eqs. (15) and (20) by the appropriate radii, there results the equation:

$$\{(M_1-M_2+M_3)+(m_2+m_3)-(m_5+m_6)\}g-\tau_{fr}/r =\{(M_1+M_2+M_3)+(m_1+m_2+m_3 +m_4+m_5+m_6)+(I/r^2)\}a, \tag{22}$$

which in light of Eq. (11) becomes

$$M_3g=\{(\text{total mass of tape and weights}) +(I/r^2)\}a. \tag{23}$$

This final equation, since it is the sum of equations of the type $F=ma$, is itself an $F=ma$ equation. It shows that the net unbalanced force is equal to the total mass accelerated plus the so-called equivalent mass of the pulley, I/r^2, times the acceleration. Note that this analysis precludes the necessity of actually measuring the friction torque and the individual masses; the acceleration can be found from the knowledge of the weight of the rider, the total mass of the system, and the equivalent mass of the pulley.

In the laboratory, the student can compare the acceleration calculated from Newton's second law with the acceleration actually found on a Cenco (Catalog No. 74935) Atwood machine. The weights are supported by a continuous waxed tape, and a synchronous spark timer makes a record of the displacement of the system as it sparks through the tape to the cylindrical pulley. From the dots on the tape the acceleration can be found by the usual methods. The hollow aluminum pulley, which rides in conical bearings, has its moment of inertia stamped on it.

Two questions on Atwood's machine which should make the student think are: Why doesn't friction appear explicitly in the final equation? and, Does doubling M_3g, the net unbalanced force, double the acceleration?

The American Association of University Women Fellowships for Women, 1952–1953

Twenty-five fellowships are offered by the American Association of University Women to American women for advanced study or research during the academic year 1952–53.

In general, the $1500 fellowships are awarded to young women who have completed two years of residence work for the Ph.D. degree or who have already received the degree; the $2000–$2200 awards to more advanced students or those who may need to study abroad; the $3000 awards to more mature scholars who need a year of uninterrupted work for writing and research. Unless otherwise specified, the fellowships are unrestricted as to subject and place of study.

Applications and supporting materials must reach the office in Washington by December 15, 1951. For detailed information concerning these fellowships and instructions for applying, address the Secretary, Committee on Fellowship Awards, American Association of University Women, 1634 Eye Street, N.W., Washington 6, D. C.

Fellowships Offered

Stipend

$1000—for research outside the United States, in eugenics and euthenics

$1000—for study in social work

$1500—fourteen fellowships, unrestricted

$1500—to a graduate of any college belonging to the Southern Association of Colleges and Secondary Schools, preferably in the field of history

$1500—for study or research outside the United States

$2000—three fellowships, unrestricted

$2000—for research in chemistry, PHYSICS, or biology (The doctorate is required.)

$2000—for study or research outside the United States

$2200—unrestricted

$3000—two fellowships, unrestricted

Comments on Several Friction Phenomena

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(Received January 8, 1951)

It is suggested that the term dry friction be limited to phenomena observed under highly outgassed conditions. Two theorems are suggested, one on the distribution of heat energy between two friction surfaces and the other on the independence of the heat loss in certain clutches, on the coefficient of friction.

Several suggestions using concepts of the physics of the solid state are made to give a better picture of the ultimate cause of friction.

THE recent review "What about friction?"[1] gives an excellent survey of the present status of the theory of friction. A part of the conclusion, "··· that there is barely a single phase of the subject which is not still in the controversial stage," is really quite surprising considering the time friction has been studied. It is the purpose of this note to comment on several aspects of friction which have appeared during the consideration of another topic.

Friction phenomena have been roughly divided into three classes: dry, boundary, and lubricated. It is suggested that more order can be introduced in discussion of friction if dry friction be limited to phenomena observed under highly outgassed or vacuum conditions. Using this definition, relatively few papers[2,3] on dry friction phenomena have been published. The term boundary friction then would refer to the conditions under which lubrication is produced by traces of oil, oxides, powdered solids, or absorbed gases. Much of the recent work will thus readily be included under boundary friction. Lubrication phenomena which are produced by an excess of oils have been extensively studied but do not form a part of this discussion. It is true, however, that no definite discontinuity exists between these three aspects of friction. Many observations, such as the failure of some bearings, suggest that all three aspects of friction may be acting simultaneously or successively.

The following theorem is suggested, for it appears to be necessary to explain the failure of an industrial clutch. *The heat developed by the friction process is apportioned between two surfaces in direct proportion to their coefficients of heat conductivity.* Some care must be observed in applying this theorem, for in a clutch where the friction facing remains in contact with the iron surface after engagement, heat may be transferred by other than friction processes. In this particular clutch the heat conduction path was so long that an excessive temperature was reached at the friction surface. The above theorem could be considered as applying at the instant of the heat generating process. Very simple measurements made on a small laboratory Prony brake indicate that the above theorem is valid. Another theorem is also suggested. *In a mechanical system in which energy is transferred from one portion of a system in motion to another part initially at rest by means of a friction clutch, the heat loss is independent of the coefficient of friction.* The proof becomes, in a sense, merely a restatement of the conservation of angular momentum.

Interesting friction phenomena may appear in very common machines of which the common bolt is an example. Some metals under extreme pressure may act as lubricants. Experiments seem to indicate that the "bolt force" developed in the common machine bolt is very dependent on the conditions of the threads. Threads plated with soft metals such as copper and silver yield much higher bolt forces for the same input torque. Under some conditions this variation in force may be a factor of from four to six. This factor is so large and, from a practical viewpoint, so variable that the phenomena probably should

[1] F. Palmer, Am. J. Phys. **17**, 181, 327, 336 (1949).
[2] Bowden and Hughes, Proc. Roy. Soc. (London) **A172**, 263 (1939).
[3] Ernst and Merchant, "Surface friction of clean metals," Proceedings of the summer conference on friction and surface finish. Massachusetts Institute of Technology (1940), p. 276.

receive more consideration. It is interesting to note that a lack of faith in torque wrenches is already appearing among the better workmen.

The ultimate cause of dry friction[4] must be found in phenomena related to the sharing of electrons between the two surfaces. There must extend from free surfaces electron densities which have surface periodicities.[5] These electron densities vary in magnitude, but are greatest directly over each atom. Thus, two materials having the same lattice constant, provided they have the same electron densities, should be expected to exhibit higher coefficients of friction. Data[2,3] to substantiate this statement are, at present, very meager. The origin of the heat developed during friction must be found in the interaction of the electron atmospheres of the atoms on the surfaces of the two solids. Electron densities, extending above the friction surfaces, become attracted by the sharing of electrons which results in the formation of bonds that are as real as those holding either atom in position. As motion proceeds, these bonds become stretched and ultimately separate at the weakest point, which need not be at the location of the original bond. It should thus appear that, even between well-lubricated surfaces when metal-to-metal contact occurs, atoms from one surface may adhere to the other surface. When rupture occurs, each atom involved becomes the source of a wave traveling into the solid where the energy adds to that already established by standing waves due to temperature (Debye's theory of atomic heat). Boundary friction, in contrast, occurs where the intensity of the electron densities has been reduced by absorbed materials.

It is suggested that in further friction studies involving metals, the surface treatment of specimens be more carefully specified and controlled. Experience indicates that many surfaces as they "wear in" experience a surface plastic flow which results in a noncrystalline layer adhering to an organized or crystalline interior. It would be expected that two such plastically deformed surfaces would exhibit lower coefficients of friction for dry and possibly for boundary friction.

[4] Tomlinson, Phil. Mag. **7**, 905 (1929).
[5] Rice and Teller, *The Structure of Matter* (John Wiley and Sons, Inc., New York, 1949), p. 180.

Let us first try to define what we are to mean by the hypothesis that physics is true. I want to adopt this hypothesis that physics is true. I want to adopt this hypothesis only to the extent to which it appeals to educated common sense. We find that the theories of physicists constantly undergo modification, so that no prudent man of science would expect any physical theory to be quite unchanged a hundred years hence. But when theories change, the alteration usually has only a small effect so far as observable phenomena are concerned. The practical difference between Einstein's theory of gravitation and Newton's is very minute, though the theoretical difference is very great. Moreover, in every new theory there are some parts that seem pretty certain, while others remain very speculative. Einstein's substitution of space-time for space and time represents a change of language for which there are the same sort of grounds of simplicity as there were for the Copernican change of language. This part of Einstein's theory may be accepted with considerable confidence. But the view that the universe is a three-dimensional sphere of finite diameter remains speculative; no one would be surprised if evidence were found which would lead astronomers to give up this way of speaking.—BERTRAND RUSSELL, HUMAN KNOWLEDGE 1948.

Studies of Transmission Zone Plates

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(Received January 10, 1951)

Methods of construction of transmission zone plates are considered. The use of the zone plate for focusing transmitted wavelengths ranging from the infrared to the very soft x-ray region is discussed. An investigation is made of the focal and chromatic properties of a zone plate to be used for visible and near ultraviolet light. Photographs of several lines of the mercury spectrum made by using a zone plate as the focusing "lens" are exhibited. The similarity between the optical properties of a transmission zone plate and a converging lens are shown by both theoretical considerations and experiment.

THE transmission zone plate for focusing visible light has provided over the past eighty years an interesting example of the laws of diffraction and interference of light. The use of such plates has been limited almost entirely to classroom instruction. However, the possibility of utilizing zone plates in optical instruments of serious design apparently has not been fully exploited.

In this paper a survey of the properties of the transmission zone plate is made with the aim in mind of pointing the way to a few practical applications. Construction of zone plates is also discussed, and a report is presented explaining some actual work done with homemade samples.

The transmission zone plate for light verifies strikingly the Fresnel-zone theory of diffraction. The plate has many of the optical properties of a lens, and these can be shown both by theory and by experiment. Zone plates may also be used to focus radar and sound waves.

The first zone plate was constructed by Lord Rayleigh in 1871, although the results of his work were not published. Soret described the zone plate in a published paper in 1875. Neither Soret nor Lord Rayleigh did much experimenting with the zone plate, however. In 1898, R. W. Wood[1] became interested in the device. He constructed zone plates by various methods and experimented with the plates using visible light.

CONSTRUCTION OF ZONE PLATE

The zone plate is so called because it is constructed following the Fresnel-zone law. The alternate zones are opaque so as not to transmit light and the remaining zones are transparent.

One way to make a zone plate is to draw concentric circles on white paper with the radii of the circles proportional to the square roots of the consecutive integers 1, 2, 3, $\cdots$. Every other zone is blacked and the paper is then photographed, the photographic negative constituting the zone plate. Because of the difficulty in drawing a large number of circles accurately, it may be desirable to use another method of construction.

Any device which gives an interference pattern of concentric circular areas which obey the Fresnel-zone law may be used for making a zone plate, providing the pattern can be photographed.

The dark rings of a Newton-rings interference pattern have radii proportional to the square roots of consecutive integers. The dark and bright rings have equal areas when parallel monochromatic light is used. Thus, it is possible to obtain a zone plate by photographing Newton's rings. Several zone plates were made by this method, using a 10-cm-diameter, double-convex lens illuminated by parallel monochromatic light rays from a mercury-vapor lamp. The lens had equal radii of curvature of 164 cm. By isolating light in the 5461A region with a filter, quite sharp rings were formed. These rings were photographed using both Kodak Process film and 649-GH Kodak Spectroscopic plates, the latter having a resolving power of approximately 1000 lines per mm. A zone plate having over 300 transparent zones was obtained on the glass spectroscopic plate, and this plate was used for test purposes.

[1] R. W. Wood, *Physical Optics* (Macmillan Company, New York, 1914), second edition, pp. 37–40, 217, 218.

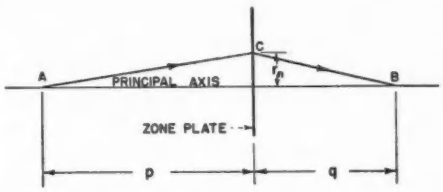

FIG. 1. Geometry of image formation for point object on principal axis at finite distance from zone plate. The object is at A, the image at B. The relationship $1/b=(1/p)+1/q$ is found to hold.

Another method of making a zone plate is by ruling the alternate zones on a transparent plate coated with an opaque substance, the ruled zones then being transparent to light.

OPTICAL PROPERTIES OF ZONE PLATE

Light rays transmitted through the transparent zones reinforce each other. This reinforcement occurs because the path from a point P on the principal axis to the second zone is one wavelength shorter than the path length to the fourth zone, and similarly for all adjacent transparent zones. An image of a source of parallel light is formed at P, and the distance b from the zone plate to P is called the focal length for a particular wavelength of light. If R_k is the distance from P to the outer boundary of the kth zone (counting both opaque and transparent zones), then $R_k=b+\frac{1}{2}k\lambda$. The radius of the kth zone is

$$r_k=(R_k^2-b^2)^{\frac{1}{2}}=[(b+\tfrac{1}{2}k\lambda)^2-b^2]^{\frac{1}{2}}$$
$$=(k\lambda b+\tfrac{1}{4}k^2\lambda^2)^{\frac{1}{2}}, \quad (1)$$

where λ is the wavelength in the same unit of length as b. For the usual case of b much greater than $k\lambda$, approximately,

$$r_k=(kb\lambda)^{\frac{1}{2}}. \quad (2)$$

The area of the kth zone is $\pi r_k^2-\pi r_{k-1}^2$, and this area is approximately $\pi b\lambda$. Actually, the areas increase slightly with increasing k, but the effect of this increase is exactly compensated for by the increase in the distance R_k. Each successive zone will contribute slightly less to the resultant intensity because of the obliqueness factor. However, we may say that the contribution of each zone is approximately the same if proper phase relationships are maintained, and intensity will be gained by increasing the number of zones in a zone plate. This conclusion holds only for the case of parallel light along the axis of the zone plate.

The Rayleigh criterion for the formation of optical images can be used to determine the allowable number of transparent zones when the point source is at a finite distance from the zone plate along the axis.[2] For the image to be sensibly perfect, the error in optical path should not exceed $\frac{1}{4}\lambda$. Consider a point source of illumination on the axis of the zone plate and at a finite distance p from the zone plate (Fig. 1), with the rays brought to focus at a distance q from the plate. The optical path is $p+q+n\lambda$, where $n=\frac{1}{2}k$ is the number of transparent zones. This path should not vary by more than $\frac{1}{4}\lambda$ for good focus, so the path length F is

$$F=p+q+n\lambda\pm\tfrac{1}{4}\lambda \quad (3)$$

by the Rayleigh criterion. Using the zone-plate formula with n instead of k,

$$r_n=(2bn\lambda+n^2\lambda^2)^{\frac{1}{2}}. \quad (4)$$

From Fig. 1 the geometrical path length is $AC+CB$, and

$$F=AC+CB=(p^2+r_n^2)^{\frac{1}{2}}+(q^2+r_n^2)^{\frac{1}{2}}$$
$$=(p^2+2bn\lambda+n^2\lambda^2)^{\frac{1}{2}}+(q^2+2bn\lambda+n^2\lambda^2)^{\frac{1}{2}}.$$

Expanding these terms by the binomial theorem,

$$F=p+q+n\lambda b\left(\frac{1}{p}+\frac{1}{q}\right)$$
$$+\tfrac{1}{2}n^2\lambda^2\left(\frac{1}{p}+\frac{1}{q}-\frac{b^2}{p^3}-\frac{b^2}{q^3}\right)+\cdots. \quad (5)$$

Neglecting terms in higher powers of $n\lambda$,

$$F=p+q+n\lambda b\left(\frac{1}{p}+\frac{1}{q}\right).$$

But from Eq. (3), approximately,

$$F=p+q+n\lambda.$$

Equating the right sides of the last two equations,

$$n\lambda b\left(\frac{1}{p}+\frac{1}{q}\right)=n\lambda. \quad (6)$$

[2] Stanley Miller, unpublished work under Contract W19-122 ac-9 of the U. S. Air Force Air Materiel Command, University of Colorado, 1949.

Therefore,

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{b}$$

as in the case of a lens.

Equating the right sides of Eqs. (3) and (5), retaining the $\frac{1}{4}\lambda$ and $n^2\lambda^2$ terms, one obtains

$$n\lambda\pm\tfrac{1}{4}\lambda=n\lambda b\left(\frac{1}{p}+\frac{1}{q}\right)+\tfrac{1}{2}n^2\lambda^2\left[\left(\frac{1}{p}+\frac{1}{q}\right)-\frac{b^2}{p^3}-\frac{b^2}{q^3}\right]. \quad (7)$$

In Eq. (7) substitute $1/b$ for $[(1/p)+1/q]$ and $[(1/b)-1/p]^3$ for $1/q^3$, giving

$$\frac{1}{4}>\left|\frac{1}{2}\frac{n^2\lambda}{b}-\frac{1}{2}n^2\lambda b^2\left[\frac{1}{p^3}+\left(\frac{1}{b}-\frac{1}{p^3}\right)^3\right]\right|=\left|\frac{3}{2}\frac{n^2\lambda b}{pq}\right|.$$

Therefore,

$$n<(|pq/6\lambda b|)^{\frac{1}{2}},$$

or

$$n<[|p^2/6\lambda(p-b)|]^{\frac{1}{2}}. \quad (8)$$

Equation (8) gives the allowable number of transparent zones for good focus when the point object lies on the principal axis. It is apparent that for the case of parallel light incident normally on the zone plate, the number of zones is not limited.

Another case to be considered is that of parallel light incident upon the zone plate at an angle. The distant point source is off-axis, so that there is an angle α between the incident light and the principal axis of a plate of n zones. One can derive an expression to calculate the maximum off-axis angle, α, at which the focus will still be good. The calculations must be made for a particular wavelength of light. The geometrical path length of a light ray is equated to the distance $b+n\lambda\pm\frac{1}{4}\lambda$. This gives the maximum allowable error in the optical path for good focus as

$$|\pm(2bn\lambda)^{\frac{1}{2}}(n\lambda/b)\sin\alpha-n\lambda\sin^2\alpha|<\tfrac{1}{4}\lambda, \quad (9)$$

if terms of $n^2\lambda^2$ and higher powers are neglected.

The resolving power of the zone plate will next be considered. It will be recalled that in the case of a lens the resolving power is obtained by computing the angle between the principal axis and the line drawn from the lens to the first minimum in the circular diffraction pattern produced by a distant point source located on the principal axis of the lens. Thus, a telescope will resolve two stars (whose light may be regarded as monochromatic with an "average" wavelength) only if their angular separation is such that the central maximum of the diffraction pattern of one star falls on the first minimum of the diffraction pattern of the other star. Similarly, one can define and compute the resolving power of a zone plate. The plate is assumed to be illuminated with rays of monochromatic light parallel to the principal axis. At the main image point on the axis all the Fresnel zones contribute their full effect with proper phase relationship for maximum intensity. But at a point slightly displaced from the axis a given transparent zone no longer contributes its full intensity, since parts of the same zone may begin interfering with other parts. Under these conditions the contribution of a given zone must be computed and then the summation for all zones must be obtained. The resulting equation, as is so often the case for diffraction problems, involves a bessel function whose first zero must be found in order that the condition for minimum intensity be established. The details of the derivation follow. From Fig. 2 the difference in paths AB and AC is

$$\Delta l\cong -b\tan\beta\sin\phi;$$

but

$$\tan\beta\cong\beta \quad (10)$$

and

$$r=r_n=b\tan\phi\cong b\sin\phi.$$

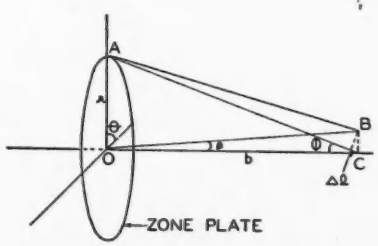

FIG. 2. Geometry of image formation for infinitely distant point object off the principal axis of the zone plate. Point B is the actual image point, whereas C is the point where the image would have been formed were the object rays parallel to the principal axis.

Therefore,

$$\Delta l \cong -r\beta \cong \pm\beta(2n\lambda b)^{\frac{1}{2}}$$

by Eq. (4). Equations (10) hold for parts of the rings in the plane of the paper (Fig. 2). For other parts, r must be replaced by $r\sin\theta$, where θ is the angle which the line from O to the point on the circle makes with the line perpendicular to the plane of the paper. If the phase caused by light from O is taken as zero, that from other parts is $(-1/\lambda)(2\pi\beta r\sin\theta)$, since the phase at C due to every point is zero. The amplitude resulting from each point is proportional to $\cos[(2\pi\beta r\sin\theta)/\lambda]=\cos(k\sin\theta)$. The amplitude resulting from a given ring is proportional to

$$\begin{aligned}
&\frac{2}{\pi}\int_0^{\pi/2}\cos(k\sin\theta)d\theta \\
&\quad=\frac{2}{\pi}\int_0^{\pi/2}\left(1-\frac{k^2\sin^2\theta}{2!}+\frac{k^4\sin^4\theta}{4!}-\cdots\right)d\theta \\
&\quad=1-\frac{k^2}{2!}\frac{1}{2}+\frac{k^4}{4!}\frac{1\cdot3}{2\cdot4}-\frac{k^6}{6!}\frac{1\cdot3\cdot5}{2\cdot4\cdot6}+\cdots \\
&\quad=1-\frac{k^2}{2^2}+\frac{k^4}{2^4(2!)^2}-\frac{k^6}{2^6(3!)^2}+\cdots \\
&\quad=J_0(k),
\end{aligned}$$

where $k=(2\pi\beta r/\lambda)$ and $J_0(k)$ is the bessel function of zeroth order. The amplitude at B is the sum of the amplitudes due to each ring or $\sum J_0\{(2\pi\beta/\lambda)(2n\lambda b)^{\frac{1}{2}}\}$, since the intensity due to each ring is approximately the same. Now

$$\begin{aligned}
&\int_0^{n-1}J_0\{(2\pi\beta/\lambda)(2n\lambda b)^{\frac{1}{2}}\}dn \\
&\qquad<\sum_1^n J_0\{(2\pi\beta/\lambda)(2n\lambda b)^{\frac{1}{2}}\} \\
&\qquad<\int_1^n J_0\{(2\pi\beta/\lambda)(2n\lambda b)^{\frac{1}{2}}\}dn. \qquad (11)
\end{aligned}$$

Letting $\{(2\pi\beta/\lambda)(2\lambda b)^{\frac{1}{2}}\}=c$, $\sqrt{n}=x$, $dn=2xdx$,

$$\begin{aligned}
\int J_0(c\sqrt{n})dn&=2\int xJ_0(cx)dx \\
&=(2x/c)J_1(cx) \qquad (12)\\
&=(2\sqrt{n}/c)J_1(c\sqrt{n}).
\end{aligned}$$

Using Eq. (12), the inequality (11) becomes

$$\begin{aligned}
&\int_0^{n-1}J_0\left\{\left(\frac{2\pi\beta}{\lambda}\right)(2n\lambda b)^{\frac{1}{2}}\right\}dn \\
&\quad=\frac{\lambda(n-1)^{\frac{1}{2}}}{\pi\beta(2\lambda b)^{\frac{1}{2}}}J_1\left\{\left(\frac{2\pi\beta}{\lambda}\right)[2(n-1)b\lambda]^{\frac{1}{2}}\right\} \\
&\quad<\sum_1^n J_0\{(2\pi\beta/\lambda)(2n\lambda b)^{\frac{1}{2}}\} \qquad (13)\\
&\quad<\frac{\lambda\sqrt{n}}{\pi\beta(2\lambda b)^{\frac{1}{2}}}J_1\left\{\left(\frac{2\pi\beta}{\lambda}\right)(2n\lambda b)^{\frac{1}{2}}\right\} \\
&\qquad-\frac{\lambda}{\pi\beta(2\lambda b)^{\frac{1}{2}}}J_1\left\{\left(\frac{2\pi\beta}{\lambda}\right)(2\lambda b)^{\frac{1}{2}}\right\}.
\end{aligned}$$

The first zero of $J_1\{(2\pi\beta/\lambda)[2(n-1)b\lambda]^{\frac{1}{2}}\}$ in Eq. (13) occurs when

$$\{(2\pi\beta/\lambda)[2(n-1)b\lambda]^{\frac{1}{2}}\}\cong 2\pi\beta r_n/\lambda=3.85 \qquad (14)$$

or

$$\beta=1.22\lambda/2r_n=1.22\lambda/d.$$

This is also the approximate zero of the quantity on the right of Eq. (13), and, therefore, of the summation in Eq. (13). Therefore, the angular half-width of the first maximum, the resolving power, is $\beta=1.22\lambda/d$. This is the same as for a lens of the same aperture.

The zone plate is inherently chromatic. The focal length is inversely proportional to the wavelength of transmitted light, as is shown by Eq. (4). Neglecting the $n^2\lambda^2$ term,

$$b=r_n^2/2n\lambda. \qquad (15)$$

The ratio $(r_n)^2/2n$ is a constant for a particular zone plate. The theoretical curve of focal length *vs* wavelength is an equilateral hyperbola. For real zone plates there may be noticeable variation from the theoretical curve. No real zone plate corresponds exactly to the zone plate treated by theory. In fact, the theory of the zone plate as ordinarily given is in itself only an approximation, so that even if the "ideal" zone plate could be constructed, one would expect slight differences between its optical properties and those which elementary theory predicts.

Tests were made on a zone plate having an effective aperture of 17.6 mm and $n=100$ transparent zones. The plate was made by photographing Newton's rings. Comparator measure-

ments showed that the radii of the zones followed the square-root law. The focal length of the zone plate was measured for various wavelengths of light in the visible and near ultraviolet regions. Provided care was taken to collimate the incident beam accurately, Eq. (15) was verified within experimental error.

The position of the image of maximum intensity for a zone plate may be called the first-order focal point. Here one Fresnel zone coincides with each actual zone of the plate. Other positions closer to the zone plate are found where a focus occurs with less intensity of illumination. At one-third the distance from the plate to the first focus there will be a focal point for which three Fresnel zones coincide with each zone of the plate. The effects of two of the three Fresnel zones cancel because of a half-wavelength path difference, and only the effect of one Fresnel zone remains. The total effect then will be a reinforcement of the light, since the light rays transmitted at each successive transparent zone have a path difference of 3λ. This causes a third-order focal point. The intensity of the third-order point is approximately one-third of that at the first-order point, since only one-third of the Fresnel zones are effective at each zone of the plate. Likewise, there will be a fifth-order focus at a distance $\frac{1}{5}b$ from the zone plate. Similar reasoning leads to the conclusion that there should be no even-order focal points. However, all the zone plates tested had a second-order focal point at approximately one-half the distance from the plate to the first focus. This was caused in part by the opaque and transparent zones having unequal areas. As a result, the effect of the two Fresnel zones, which should coincide with each actual zone of the plate at the second-order focal point, do not completely cancel, and a partial reinforcement of light occurs.

There are also virtual foci which occur along the principal axis at distances $-b$, $-\frac{1}{3}b$, $-\frac{1}{5}b$, and so on, where b is the first-order focal length of the zone plate.

EXPERIMENTS WITH ZONE PLATES

The optical properties of a real zone plate were investigated experimentally to find its advantages and disadvantages when used as a lens. A narrow slit illuminated by a quartz mercury-vapor lamp was used as the source of monochromatic light. The light was made parallel by a quartz lens, then dispersed by a plane diffraction grating so that different wavelengths could be selected for focusing by the zone plate.

In this manner, images of certain spectral lines were formed by the zone plate and these images were recorded on 35-mm Super XX film. The Newton-ring zone plate used in the tests was a Kodak 649-GH glass spectroscopic plate

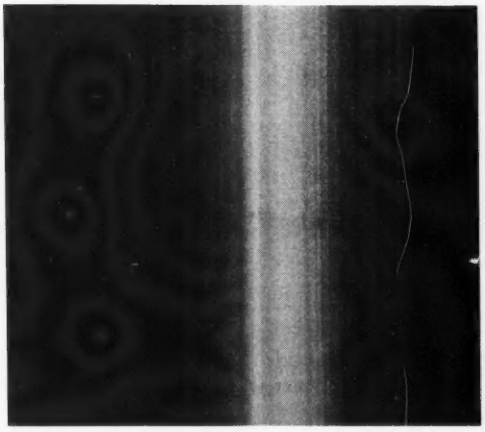

Fig. 3

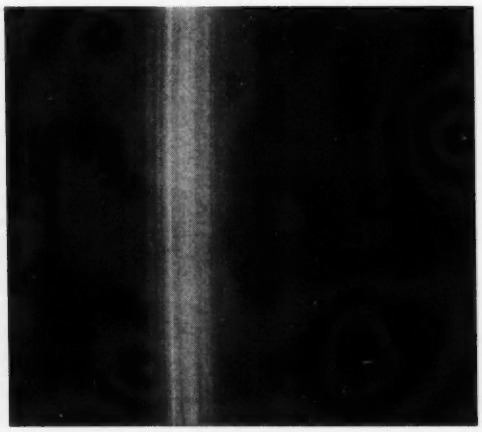

Fig. 4

FIGS. 3 and 4. The image of the 5461A line of mercury at the first- and second-order focal points, respectively. The focal lengths were 66 cm and 33 cm as measured from the zone plate to the photographic film.

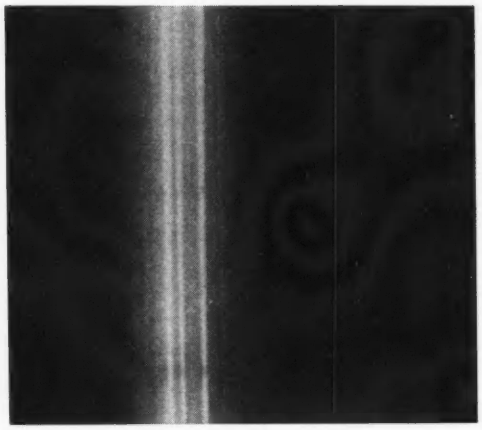

Fig. 5

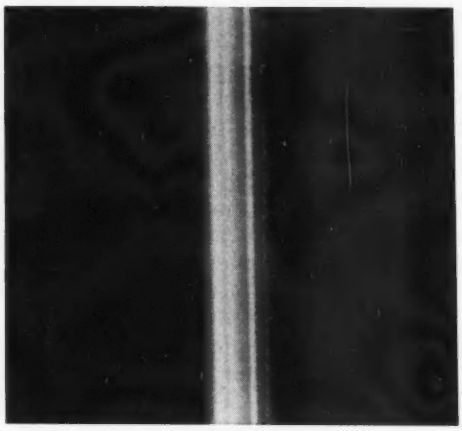

Fig. 6

FIGS. 5 and 6. A comparison of an image obtained with the zone plate (Fig. 5) and an image from a meniscus lens of the same aperture (Fig. 6). These are the triplet lines of mercury having wavelengths of 3663, 3654, and 3650A.

of 0.8-mm thickness which would not transmit wavelengths below about 3000A. Photographs were taken of spectral lines in the 3400–3600A region of the ultraviolet, as well as throughout the visible region.

The third-order focus for the line shown in Figs. 3 and 4 was not at all sharp. This is generally the case for any zone plate, since the lower orders of the same line pass through the image. Also, the chromatic separation of third-order images of all wavelengths is only one-third as great as for first-order images, which causes a fringing effect when the incident light consists of more than one wavelength.

The background line-pattern present on Figs. 3, 4, and 5 seemed to be a combination of diffraction effects produced by the narrow slit and plane grating and interference effects between different wavelengths transmitted through the zone plate. The background lines were not prominent enough to be seen by direct visual observation of the images. When a filter was

Fig. 7

Fig. 8

FIGS. 7 and 8. Aberration when a double-slit source illuminated by monochromatic light was moved off the principal axis of the zone plate. In Fig. 7 the slits are 2.9° off-axis; in Fig. 8 they are 4.1° off-axis.

used and the grating removed, the background of the image was quite clear, as in Figs. 7 and 8.

For the comparison photographs (Figs. 5 and 6), the lens and zone plate were used at an aperture of 30 mm. The zone-plate image (Fig. 5) was formed at a focal length of 85 cm, the lens image (Fig. 6) at 90 cm. The required exposure times for Figs. 5 and 6 were 8 sec and $\frac{1}{2}$ sec, respectively. In effect, then, the f:28 zone plate was approximately equivalent to an f:120 lens.

An aberration test was made using a double slit, illuminated by the mercury lamp, as the source. No grating was used, but instead a No. 77 Wratten filter for the 5461A green line of mercury was placed between the slits and zone plate. Parallel light was allowed to fall directly upon the zone plate. The slit combination was moved off the principal axis of the zone plate in a direction perpendicular to the axis in order to determine when the image became fuzzy or distorted. The photographic film remained in the focal plane of the zone plate at a 66-cm focal length. Figure 7 shows the image of the slits when the latter are 2.9° off the axis. Figure 8 shows the image with the slits 4.1° off-axis; here the lines are noticeably fuzzy. By repeating the aberration test several times the maximum off-axis angle for good focus was determined as 3°18′. The theoretical maximum angle was calculated from Eq. (9) to be $\alpha=3°15'$, using an independently determined effective aperture of 17.6 mm (100 transparent zones).

ZONE PLATES FOR DIFFERENT PURPOSES

The construction of a zone plate for visible and near ultraviolet light (similar to the plate tested experimentally) is not difficult because the number of zones per centimeter is not too great to be resolved photographically. A zone plate suitable for focusing very soft x-rays must have a much larger number of zones per centimeter in order that the focal length be reasonably short. A plate with a diameter of 0.4 cm and having 2000 transparent zones could be used for a limited range in the 10A region. This plate would have an average of 10,000 zones per centimeter and a maximum of about 17,000 zones per centimeter at the outer edge, which would make the construction extremely difficult. The focal length is 100 cm for $\lambda=10$A. However, two identical zone plates spaced a small distance apart could be used to give an equivalent focal length approximately one-half the individual focal lengths, just as in the case of two identical lenses.

TABLE I. Comparison of properties of zone plate and lens.

Property	Converging lens	Transmission zone plate
Chromatism	Present	Pronounced
Image forming	$\frac{1}{f}=\frac{1}{p}+\frac{1}{q}$	$\frac{1}{f}=\frac{1}{p}+\frac{1}{q}$
Higher orders	None	1, (2), 3, (4), 5, ⋯
Resolving power	$\beta=1.22\lambda/d$	$\beta=1.22\lambda/d$
Focal length	$\frac{1}{f}=[\mu(\lambda)-1]\left(\frac{1}{r_1}+\frac{1}{r_2}\right)$	$\frac{1}{f}=\frac{1}{r_n-n\lambda}-\frac{1}{r_n+n\lambda}$

A zone plate for use in the infrared region around 25,000A might have the constants $n=800$ transparent zones and $r_n=4.52$ cm. Then the focal length is $b=50$ cm for $\lambda=25{,}000$A, calculated from Eq. (4).

A summary of the optical properties of the zone plate in comparison with a converging lens is given in Table I.

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After I began teaching it in the fall I wrote home to my parents that I had been exceedingly lucky in being assigned to the teaching of physics, rather than of language or mathematics, because physics was a subject which taught itself.—ROBERT A. MILLIKAN, *Autobiography.*

Demonstration of Phase Shift Using Voltmeters

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(Received November 22, 1950)

Demonstration of phase shift, attenuated waves, and some properties of oscillators is made with the aid of direct current voltmeters connected to various points in a one-tube phase shift oscillator circuit. Values of circuit components are given for a simple, stable oscillator with a period of the order of 10 to 15 seconds.

PHASE difference between electric oscillations of the same frequency is usually demonstrated with the help of an oscilloscope and a phase shifting network. Although undoubtedly very instructive of Lissajous figures, it is also true that this method requires considerable interpretation by the student if the concept of phase difference is to be abstracted from a stationary ellipse or circle on the screen. Use of an electronic switch to display the two oscillations on the oscilloscope simultaneously is more flexible, but suffers from the same disadvantage in that it is a stationary pattern which is viewed.

To give a direct demonstration of phase difference, attenuated waves, and several basic principles of oscillators, a phase shift oscillator[1–3] of frequency of the order of 0.1 cycle sec^{-1} has been constructed (Fig. 1). In order to obtain reliable operation at such a low frequency, excessive degeneration must be avoided. This is the reason for the cathode bias arrangement and for the use of voltage regulator tubes. The oscillation takes place at that frequency for which the total phase shift from A to D is $+180°$, approximately a $+60°$ shift in each section of the network. Standard 20,000-ohm/volt dc voltmeters, of 250-volt range, are clipped between points A and A', B and B', and C and C'. It is desirable to offset the zeros of the latter two voltmeters with a B-battery, as shown, or by other means. Upon proper adjustment of R_3 (feedback control) and R_4 (bias), a nearly sinusoidal oscillation builds up in the system. The reading of the voltmeter at A varies between about $+20$ volts and $+220$ volts. The loading effect of the voltmeters is not serious. For the values of R and C shown, the period T is about 10 sec, which is a convenient value for demonstration purposes. Periods of 20 sec and longer may be realized by increasing the product RC.

The voltmeters follow the oscillations, and the phase shifts are readily apparent, as is the decrease in amplitude in each section of the network and consequent necessity for the amplifier tube. Using a vacuum-tube voltmeter, it is also possible to demonstrate the fact that the grid voltage is exactly out of phase with the plate voltage. Various clipped wave forms may be produced by varying R_4, with R_3 at its maximum value, illustrating distortion in an overdriven oscillator and limitation of amplitude due to tube characteristics. The build-up of oscillation and the effects of transients are easily seen.

The arrangement of Fig. 1 gives a wave traveling from C to A with *increasing* amplitude and with phase differences of about 60° between successive points. By reversing the connections of voltmeter B and returning it to a positive zero offsetting point (obtained from a bleeder across B-plus), a more conventional wave is obtained which travels from A to C with *decreasing* amplitude and which has phase differences of about 120° between successive points.

It is an instructive problem in ac networks to derive some of the theory of the symmetrical oscillator for which $C_1=C_2=C_3=C$ and $R_1=R_2=R_3=R$. Usually $R_L \ll R$; this amounts to saying that the current through C_1 is a negligible fraction of the plate current. Under these circumstances, oscillation takes place when

$$-GRi_3=V, \tag{1}$$

where G is the voltage gain of the tube, i_3 is the

[1] E. L. Ginzton and L. M. Hollingsworth, Proc. Inst. Radio Engrs. **29**, 43–49 (1941).
[2] W. C. Kunde, Electronics **16**, 132–133 (November, 1943).
[3] H. P. Richter, *Fundamentals of Industrial Electronic Circuits* (McGraw-Hill Book Company, Inc., New York, 1947), pp. 384–385.

current through R_3, and V is the potential difference between point A and ground. The minus sign embodies the condition of 180° phase shift in the network. Using complex representation of the impedances, Kirchhoff's laws are applied to solve for i_3 in terms of V, R, C, and ω; then Eq. (1) can be satisfied only for one frequency, which is the frequency of oscillation. Results are: $T=2\pi 6^{\frac{1}{2}}RC$; $V_B=0.306V$ at $+55.8°$; $V_C=0.0912V$ at $+112.2°$; $V_D=0.0345V=V/29$ at $+180°$. The voltage gain of the tube must be $\geqslant 29$ for sustained oscillation.

In case R_L is not negligible, Eq. (1) must be replaced by $-GRi_3=V+i_1R_L$, where i_1 is the current through C_1. It is assumed that the tube is a pentode for which $G=R_Lg_m$. The resulting period and minimum gain are both increased: $T=2\pi(6+4R_L/R)^{\frac{1}{2}}RC$, and $G=29+23(R_L/R)+4(R_L/R)^2$.

The required gain may be reduced well below the value of 29 by several means, thus permitting the use of a triode tube. For a four-section symmetrical oscillator,[1] $G=18.2$. A tapered oscillator has been described[4] in which $R_2=aR_1$, $R_3=a^2R_1$, and $C_2=C_1/a$, $C_3=C_1/a^2$, and similarly for a four-section tapered oscillator. For $a=3$, $G=12.85$ for three sections and $G=6.5$ for four sections. These modifications are not very useful at low frequencies, however, since excessively large capacitors would be needed. For instance, C_1 for a three-section tapered oscillator is 11.3 times as large as for a three-section symmetrical oscillator. Likewise, the four-section symmetrical oscillator involves larger capacitors than does the three-section symmetrical oscillator.

For $R=1$ megohm, $C=0.5$ μf, and $R_L=0.27$ megohm, the calculated period is 8.4 sec. The measured period was from 9.4 sec to 14.1 sec, depending upon the paper capacitors which were used. For this measurement, only voltmeter A was connected, which provides negligible shunting of R_L. Leakage and dielectric absorption thus seem to play an important role in determining the period at very low frequencies. If each capacitor is shunted by a resistance r, where $r\gg R$, the computed period is increased by a factor $(1+5R/r)$; this ratio was quantitatively verified using $r=40$ megohms to shunt each capacitor. The "leakage resistance" of each capacitor was determined by the ballistic method, using an initial potential difference of 255 volts. The capacitors which gave the longer periods of oscillation showed "resistances" of from 36 to 700 megohms, while those giving the shorter period had "resistances" of the order of 5000 megohms. The correlation between period and leakage was, therefore, qualitatively correct, but quantitative agreement with simple theory was not found.

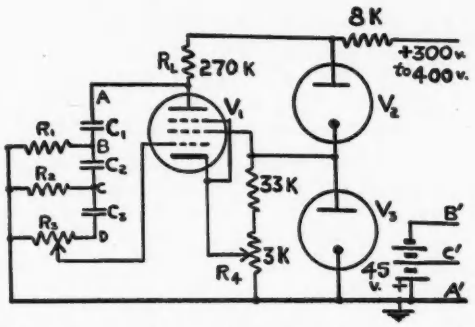

FIG. 1. Phase shift oscillator. R_1, R_2, R_3, 1 megohm; C_1, C_2, C_3, 0.5 μf; V_1, 6AC7 or 6SH7; V_2, VR150; V_3, VR105. Period of oscillation, approx. 10 sec.

The oscillator described is easily constructed, reliable in operation, and provides demonstration of some basic principles of wave motion and oscillation in an easily grasped form, since variations of potential are presented directly as readings on a familiar device, the voltmeter.

[4] P. Sulzer, Proc. Inst. Radio Engrs. **36**, 1302–1305 (1948).

Historically, the thesis can be maintained that more fundamental advances have been made as a by-product of instrumental (i.e., engineering) improvement than in the direct and conscious search for new laws.—ROBERT A. MILLIKAN, *Autobiography.*

The Electron Theory of Solids*

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(Received January 31, 1951)

This article is a brief review of the way in which the original ideas of Lorentz and Drude on the electron theory of solids have been expanded to cover the quantum theory of the electron gas, the energy band theory, with applications to the distinction between conductors, semiconductors, and insulators, and to treat not only electrical conductivity, but mechanical, thermal, and magnetic properties of solids, and a wide range of phenomena.

LORENTZ and Drude are generally regarded as the founders of the electron theory of solids. They came at the end of the period when physicists were willing to take such things as the values of the electrical conductivity, the dielectric constant, or the index of refraction of a substance as constants of nature, to be measured accurately and set down in a table, and at the beginning of the period when such quantities were regarded as things to be explained in terms of the electronic or atomic structure of matter. To a considerable extent they were the originators of this new view. They came at the end of the nineteenth, the beginning of the twentieth, century, and this was the time when the great change in point of view in physics came about, the change which has led to the phenomenal advances of the present century. What I wish to show in this talk is the gradual way in which the original steps taken by Lorentz and Drude have been broadened in the succeeding years, so that now we are no longer satisfied with merely explaining the electrical properties of solids by the electron theory, but we find that we are irresistibly led by our study to a complete investigation of all the properties of a solid, its mechanical and chemical and thermal properties just as much as its electrical and magnetic and optical ones. This broadening of the aim of solid-state theory has resulted in our still being far from the complete goal of understanding all about the structure of a solid, but it has increased enormously the interest and fascination of the problem.

The model of a solid which Lorentz and Drude used was simple, but in many ways very adequate. They assumed that the atoms of a dielectric contained electrons held to a position of equilibrium by a restoring force proportional to the displacement, so that they could oscillate with simple harmonic motion, and resonate in an external electromagnetic field. This simple model led to the observed type of formula for dielectric constant, and its variation with frequency, resulting in anomalous dispersion and the behavior of the index of refraction. In a metal, they assumed as well that there were free electrons, capable of wandering through the lattice, colliding with the atoms as they went. This hypothesis led to electrical conductivity following Ohm's law. It also led to a dependence of the conductivity on frequency which agreed with observations on the index of refraction and absorption coefficient of metals at high frequencies: at high enough frequency, the metal no longer absorbs as much radiation as its conductivity at low frequencies would suggest; and, in fact, in the far ultraviolet and x-ray region it is no more opaque than an insulator, and the theory explained this. The hypothesis led as well to a theory of thermal conductivity by the free electrons, and predicted correctly the Wiedemann-Franz ratio between electrical and thermal conductivity. The model of the atomic oscillators led to a simple theory of the Zeeman effect, the Faraday effect, and the other relations between magnetic and optical properties. These were great triumphs for the electronic and atomic point of view.

There were several conspicuous gaps in the success of the theory, however. It explained Ohm's law in terms of the resistance which is equivalent to the collisions which the electrons

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make with the obstacles in the form of atoms which they strike in passing through the crystal. But it failed to explain the experimentally simple fact that the resistance of a normal metal decreases linearly to zero at the absolute zero of temperature. What possible mechanism could lead to the disappearance of the obstacles to the motion of electrons, as the temperature went down? Then again, it explained thermal conductivity, but failed completely to explain why the free electrons of a metal do not experimentally contribute to the specific heat, as a free electron theory would lead us to believe they should. How can the electrons possibly partake in thermal agitation, as they clearly must if they can transport thermal energy, and yet not have any specific heat? Again, it could give a simple explanation of the Hall effect, according to which a transverse difference of potential is set up across a conductor carrying a current in a transverse magnetic field, and it gave a simple calculation of the Hall coefficient, which indicated that it should always have the same sign, since it came out proportional to the charge on the electron, which was always negative. But some metals experimentally have a Hall coefficient of the opposite sign. Coming to more fundamental questions, it could explain why a metal should have conductivity if it had free electrons, but it failed to explain why metals had free electrons, and insulators did not, and failed to explain the nature of semiconductors, intermediate between metals and insulators.

The next significant advances in the theory came with the quantum theory, and are connected with the names of Fermi, Dirac, and Sommerfeld. They came from the application of quantum statistics to the free electron model. Classical statistical mechanics operates with a phase space, a space in which the coordinates and momenta of a particle are plotted as variables. In the classical theory of a perfect gas, it is shown that at a given temperature the density of molecules in this phase space is given by the Boltzmann factor $e^{-E/kT}$, where e is the natural base of logarithms, E the energy of the molecule (a function of its coordinate and momentum, or of position in the phase space), k is Boltzmann's constant, and T the absolute temperature. It is this distribution law which leads to the equipartition of energy, and to the classical prediction as to the specific heat of the electrons in a metal, which was not in agreement with experiment. Fermi and Dirac, however, discussed a perfect gas according to quantum statistics. In the first place, quantum statistics leads to an identification of volumes, or cells, in phase space, with the stationary states of quantum theory. These cells have a volume of h^n, where h is Planck's constant, n the number of coordinates concerned in the problem. Heisenberg's uncertainty principle states that a molecule cannot be localized in phase space more closely than saying that it is located in one of these cells; and the whole essence of quantum theory is that the most accurate description of nature which we can give is to say how many molecules are in each of these cells, which are identical with stationary states. If now we are dealing with electrons, which obey Pauli's exclusion principle, which states that there can be no more than one electron of a given spin in a given stationary state, we see at once that the Boltzmann distribution law cannot be right, since it allows densities of greater than one per cell in the phase space. Fermi and Dirac showed that this law was to be replaced by the Fermi distribution law, well known to all, which reduces to the Boltzmann distribution at high energies, where the average number of molecules per cell is small compared with unity, but approaches a maximum value of unity at low energies.

Sommerfeld at once seized on this distribution law, and saw that it provided the explanation for one of the puzzles of the Lorentz-Drude theory, the fact that the free electrons of a metal experimentally do not show any specific heat, and yet can carry thermal energy. The explanation is simple in terms of the Fermi-Dirac statistics. In a metal at ordinary temperature, almost all (all but a fraction of a percent) of the electrons are in stationary states which are filled to their maximum allowable density. An increase of temperature makes no change in the energy of these electrons in filled states; its only effect is to increase slightly the energy of those electrons at the top of the distribution. Thus the only contribution to the specific heat is provided by a very small fraction of the electrons, and is almost negligible, though not strictly zero. But

it is just these electrons at the top of the distribution which are effective in transporting heat; and Sommerfeld showed that they were not only able to carry the heat, but that the theory led to a calculation of the Wiedemann-Franz ratio in agreement with experiment, just as the Lorentz-Drude theory had done.

There were several puzzles still left, however, in particular the temperature variation of the conductivity, and the Hall effect, as well as the fundamental differences between conductors and insulators. The main answers to these puzzles were all provided in rapid succession after the work of Sommerfeld. Houston explained the temperature variation of conductivity by making the observation that the electrons moving freely through the crystal really had to be treated by wave mechanics, and that their wavelengths were in most cases long compared with the interatomic distance. Now the behavior of such waves is known from many branches of physics. For instance, in optics, a light wave of wavelength long compared with the interatomic distance passes through a crystal without scattering, except by the irregularity of density resulting from impurities, or from atomic vibration; the theory of scattering arising from density fluctuations had been worked out long before. Houston showed that the same situation arose in a metal, and that at the absolute zero of temperature the electron waves traversed it without scattering, and hence with infinite mean free path, whereas at higher temperature the lattice irregularities resulting from temperature agitation produced a scattering, or a resistance, proportional to temperature, in accordance with observation.

In some metals the wavelengths of the free electrons are large compared with the interatomic distance, as Houston assumed; but in others they are comparable, and then we must expect another effect to enter. This is the Bragg scattering of the electron waves by the atomic planes, just as x-rays are scattered when their wavelengths are comparable to interatomic distances. The developments of which I am now speaking took place in the neighborhood of 1928, and electron diffraction had just been discovered by Davisson and Germer, and its theory had been discussed in detail by a number of physicists, including Bethe. Bethe's theory followed the lines of the dynamical theory of x-ray diffraction set up a decade earlier by Ewald and others, and treated the interaction of the direct and reflected waves by wave mechanics, as did several other writers at about the same time. Bethe did not make connections which now seem obvious to us between his theory, dealing with electrons of high energy able to emerge from the crystal, and the scattering of electrons at the top of the Fermi distribution. Bloch, however, at just the same time, investigated the motion of electrons in a periodic lattice from the standpoint of Schrödinger's equation, starting with electrons bound to individual atoms, and treating the interaction of the atoms as perturbations. He showed that the energy levels of the individual atoms broadened under the action of the perturbations into energy bands, and that each wave function in one of these energy bands was characterized by a wavelength, or a momentum related to it by de Broglie's condition. It became evident to physicists at once that Bethe's problem and Bloch's complemented each other: each was treating the motion of electrons in a periodic potential, Bethe by a perturbation process starting with a free electron as the starting point, Bloch starting with the bound atoms, but each coming to similar general theorems relating to the wave functions and energy levels. In particular, according to Bloch's scheme the energy levels broadened into bands, leaving values of energy between the bands, or gaps, in which there were no energy levels. Bethe started with all energy levels allowed, as they are for a free electron, but showed that the perturbation introduced certain gaps, or forbidden bands, of energy, coming for those particular electrons which satisfied the Bragg scattering conditions to a good approximation, and therefore could not be propagated freely through the lattice. In this way a close connection was introduced between electrical conductivity and electron diffraction, which would hardly have been suspected before.

This theory of energy bands, originated by Bethe, Bloch, and others, and developed by Brillouin, soon led at the hands of A. H. Wilson to the explanation of the difference between metals, insulators, and semiconductors. It was

shown by Bloch that an energy band completely filled with electrons could not carry current under any circumstances. Thus, a substance all of whose electrons were in filled bands, with an energy gap immediately above the top filled band, would necessarily be an insulator, while one with electrons partly filling a band would be a conductor. Wilson showed that the semiconductors were the intermediate case, in which electrons filled a band as in an insulator, but with a narrow gap above, with often certain impurity levels in the gap, so that thermal excitation could raise electrons to the otherwise empty level, leading to conductivity which would increase rapidly with the temperature, as the number of available electrons increased from thermal excitation. Thus, the old problem of why some substances were conductors, some insulators, yielded to a theoretical concept derived from the theory of electron diffraction, another example of the way in which the scope of the theory of solids has constantly increased and has uncovered new relations between phenomena which at first seemed unrelated. The explanation of the two possible signs for the Hall effect also came out in this connection: a full band from which a few electrons were removed by thermal excitation or other cause was shown to conduct current as if it contained positive rather than negative electrons, and these would show the positive Hall effect. With these discoveries of the early 1930's the foundation was laid for the great experimental developments in the field of semiconductors of the last few years.

Once the importance of the energy bands was established, naturally there was an effort to calculate them from first principles. It was at once realized that they formed the extension, to the theory of solids, of the method of self-consistent fields introduced for atomic problems by Hartree a number of years earlier. In a periodic potential, such as one has in a solid, the solutions of Schrödinger's equation take the form of modulated plane waves, leading to a periodic charge density. The self-consistent solution states that the potential arising from this charge density must equal the originally assumed potential. For the atomic problem, Hartree had shown that it was possible to solve this self-consistent field problem; it was rendered feasible largely by the spherical symmetry, which introduced great simplifications. Wigner and Seitz pointed out that some of the same simplification could be achieved in a solid, particularly in a metal, by noting that at least in the neighborhood of each atom, the potential is very nearly spherically symmetrical, so that Schrödinger's equation can be solved near each atom as in a spherical problem, and the solutions could be pieced together to get the proper periodicity from one atom to another. By the work of Wigner and Seitz, extended by the present writer and others, it was possible to calculate energy bands for a number of types of solids, both metals and insulators, and to show that the theory predicted correctly which should be metals, which insulators. It did more than this, however. It allowed one to find the energy levels of the electrons as a function of internuclear distance, assuming the crystal to be allowed to expand or contract, and from this, by quantum mechanics, one could calculate the energy of the whole crystal as a function of internuclear distance, and hence estimate the lattice spacing under equilibrium conditions, and the compressibility. Thus, there was another great extension of the scope of the electron theory of solids: it became a recognized aim to explain the crystal structure and lattice energy, and hence all the mechanical properties, as well as the electrical behavior.

The understanding of the nature of energy bands made it possible to do some other things which had not been tried before, as well as to improve calculations that had been made with moderate success. An example of the latter was the calculation of optical constants of solids, where Kramers and Heisenberg had shown that the optical resonance behavior of the bound electrons could be explained by quantum mechanics, and where the free electrons proved in quantum theory to behave much as in the classical theory, as far as optical behavior was concerned. An example of the new possibilities opened by energy-band theory was in the explanation of the x-ray absorption and emission spectra of solids. It was known experimentally that the x-ray absorption spectra of metals showed a fine structure for several hundred volts above what would be the absorption edge if we

were dealing with single atoms. Kronig showed that this fine structure could be correlated with the band structure of the energy bands. Similarly, the emission of x-rays from the conduction band of metals to lower x-ray levels has been found to give good experimental evidence of the shape and width of this conduction band, and hence to give some of the most graphic experimental proofs of the reality of energy bands. Again, detailed study of photoelectric emission from surfaces of various sorts has given experimental information about the allowed and forbidden bands of energy. From many sorts of experiments, we are coming to feel that the energy bands have a definite experimental reality.

All these types of experiments and theories are alike in one respect: they do not depend on the so-called exchange effects, which arise on account of the effect of electron spin on electronic interaction, through the medium of the exclusion principle. The things which we have spoken of are analogous to the information about atomic spectra which we should have if we considered only the electronic configurations of the atoms, and disregarded the multiplet structure. We know the general nature of these exchange effects in atoms. For instance, the exchange has the effect not only of splitting up a given atomic level into multiplets, but of arranging these multiplets so that that corresponding to the greatest multiplicity, or to having the maximum possible number of electrons with parallel spin, has the lowest energy. The theories of these effects for atoms and for simple molecules were being worked out by Hund and the present writer, by Heitler and London and others, at the same time as many of the developments of which we have been speaking; and Heisenberg very early in the game made the very fruitful suggestion that these effects probably were the explanation of ferromagnetism. It has been a puzzle, ever since the time of Weiss, how the Curie point of a ferromagnetic material could be so high. What form of interaction energy between the elementary magnets forming a solid could be so great that temperatures of hundreds of degrees were required to disturb their orientation? The magnetic energies of interaction between them, which could be easily calculated from their known magnetic moments, would be thousands of times too small. Heisenberg pointed out that the exchange interactions, which formally acted like torques between the magnets, as shown particularly by Dirac and Van Vleck, were of the right order of magnitude to explain ferromagnetism. Heisenberg's argument was stated in the language of the Heitler-London theory, which is very satisfactory for the hydrogen molecule, but has great disadvantages for dealing with solids. Bloch, however, showed that a free electron type of theory led to similar results, and the present writer and Stoner developed this into an energy-band theory of ferromagnetism, based according to Heisenberg's suggestions on the exchange effect, but making closer connection with the other properties of metals. Here again, the scope of the electron theory of metals broadens, and includes magnetic phenomena as things which must be explained and understood.

Let us ask where this leaves us at present with the electron theory of solids. Most of the developments we have been speaking of took place before the war; most, in fact, in a period of five or six years following 1928. Activity then fell off; many of the leading workers, whose names have been mentioned in this survey, dropped out of the field, and the opinion was expressed that all the interesting parts of the theory had been worked out. Is this true? I believe, returning myself to the field after being out of it for a number of years, that this is not at all the case; and I am interested to see a number of other physicists going back to the theory of solids too. It is true that many things have been explained, and that probably there will not be another period of as fast and striking advance as that from 1928 to 1934. But a great many of the things that were worked out in that period were done very hurriedly, without examining the interrelationships between the various problems. Just as an example, we know now that a properly computed set of energy bands for a certain metal should explain its electrical properties, its mechanical behavior, its magnetism, its x-ray absorption and emission, its electron diffraction properties, and so on. And yet there is no single substance on which nearly enough experiments have been made, or theory worked out, to give any sort of check to these expectations. One

thing that is needed, then, is a careful program of theory and experiment, to give real tests to the adequacy of our theoretical understanding. In a few semiconductors—germanium, silicon, the materials of the oxide cathode—extensive experimental programs are under way, on account of the importance of the materials. We should see more of these programs, on more materials, and we should see theory to match.

But apart from these more careful checks, which will become increasingly important as the theory gets quantitative enough to be of real use in understanding experimental facts, there are a number of very fundamental points in which the theory is far from adequate. We do not have any way of solving the periodic potential problem, the fundamental Schrödinger equation for the crystal lattice, which is nearly accurate enough for a satisfactory theory. And perhaps most important, we are far from having an adequate technique for taking care of the exchange problem, the whole question analogous to the multiplet theory of atoms. The Heisenberg method, based on the Heitler-London scheme, has been widely advocated. Heitler-London methods as applied to metallic structure have been made the basis of Pauling's theory of metallic behavior. The recent theories of antiferromagnetism have been based on similar arguments. And yet the fundamental basis of these applications is very dubious. Almost all such methods are based on neglect of certain terms arising from nonorthogonality of wave functions, which are actually of controlling importance. There is a very sharp division of those who try to make calculations of the behavior of molecules or solids, into two schools: one school, based on approximate application of the Heitler-London methods, which makes broad generalizations without supplying the mathematical justification for them; the other, carrying through a conscientious calculation of all the terms neglected by the first school, finding that these terms are so important that if any one of them is neglected the whole agreement with experiment is lost. The work of this second school is receiving far less attention than that of the first, for it is less spectacular, and seems to depend more on numerical calculation, less on attractive generalizations. Nevertheless, I believe that there is no question but that it is sounder, and that real advance in understanding must come in that direction.

Just to give a little variety in closing, may I go further in suggesting that the electron theory of solids is not a finished subject, and point out that one of our most simple and intuitive ideas about electricity in solids is really completely unjustified? This idea is the concept of the electric field. All of our electromagnetic theory is based on the assumption that we can define an electric and magnetic field; and this is as true of the most sophisticated quantum electrodynamics as it is of the old-fashioned variety. And yet what really goes on in a crystal is that the force on an electron in that crystal, which is about the only thing we could mean by the electric field there, is not a function merely of the position of the electron, but of its motion and the direction of its spin as well. This is a result of the fundamentals of Schrödinger's equation, and comes out very straightforwardly in any approximate solution of that equation, for instance by the self-consistent field using the Hartree-Fock equations. It comes about because a rapidly moving electron does not sweep other electrons out of its way nearly as much as a slowly moving one, and because on account of the exclusion principle an electron sweeps electrons of the same spin more effectively than those of the opposite spin, so that the other electrons exert a force which depends on how fast the electron in question is moving, and what its spin is. The only way in which we can define an electric field in a solid in a microscopic way (as opposed to the macroscopic definition from the force on a stationary electron in a large cavity in the solid) is by taking a certain sort of average of the potential set up by the Hartree-Fock method; and this is certainly a rather sophisticated concept to replace our simple intuitive ideas, which simply do not have the capability of being stated in a precise way. And the magnetic field is much worse. We can define the static magnetic field in a microscopic way without much more trouble than the electric field; but if we try logically to say how we shall define an electromagnetic radiation field in a solid, we find ourselves in the middle of the complications of quantum electrodynamics, and cannot give answers of any de-

gree of simplicity or completeness to problems of a quite everyday nature, which Lorentz and Drude were accustomed to ask and to answer without any qualms. The electron theory of solids has come a long way; but it has not yet nearly reached a state which its founders would consider finished, if they were to come back and examine it. There is plenty more to do.

Suggestions for the Improvement of Physics Teaching in Colleges and Universities

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Possible defects of current physical science and physics courses for nonscience students are discussed. A solution based on the desirability of common intellectual experience for all first-year college students is proposed. Emphasis is laid on the necessity for creative laboratory experience, informal discussion in small classes, and intensive rather than extensive coverage of material for student enjoyment and appreciation of science.

THE science of physics seems to stand in an anomalous position these days. On the one hand, it is lauded for its exposure of Nature's secrets for the use and probable benefit of mankind. On the other hand, it is considered too difficult for even the nonscientific *college* student to grasp and is becoming an ever smaller part of his knowledge. The more important physics becomes in its effect on our lives, the less knowledge of it we have available in deciding how to use it for better living. It is this tendency toward minimization of all sciences in liberal arts college curricula which is discussed in this article. Certain unfavorable aspects are analyzed and a solution to the problem proposed.

SELECTING THE COURSE

In most colleges and universities there seems to be agreement on using the first two of the usual four years to provide a broad general background for the student, a common fund of knowledge for all students. To this end the student is usually restricted in his course selections to choices within several broad fields. From almost any point of view these restrictions are sound and have been well supported by eminent educational authorities. But too often the selection in science is a choice between two kinds of course, either a "physical science" course or a preprofessional science course. This statement must not be taken to mean that either type of course is bad when viewed as an isolated structure. But viewed as part of the educational whole, neither course properly serves the needs of general education.

I believe that these two types of courses resulted from the good physics teacher's realization that the second type of course, the preprofessional, did not fulfill the educational need of the nonscience student. Admittedly, his realization was frequently the result of suggestions from authorities believing in general education. So a new course was built, usually by eliminating mathematics and reducing content. But by this reorganization of the general physics course into a general education course, the physics teacher reduced the amount of time students spend on his subject to a semester, or less. We have the queer result that a reduced physics course is offered just when physics is becoming widely known for the first time! And, of course, this reduction is true of all sciences studied in the physical science courses. In fact, in science several fields are frequently taught in one course; in the humanities and social sciences, rarely. (This statement is not universally true, since several engineering schools and a few universities have excellent general courses in the humanities and social sciences.)

Let us consider several concrete objections to the physical science courses as now generally taught. Even if we could grant that it is reasonable that the study of science should be the minutest part of most students' academic work,

we can find several other important objections to the physical science course. After studying these objections, we will note why the preprofessional course is also inadequate in general education.

The first objection can be stated this way. The physical science course is taught *too early*. That is, the usual physical science course seeks to provide a breadth of perception by freshmen that is rarely acquired by seniors. Not only that, nonscientific students with slight training in quantitative thinking, intellectually undisciplined, are suddenly asked to think rationally, quantitatively, and precisely about subjects quite far removed from their everyday experience. How can they possibly be expected to appreciate the beauty of the work of Newton, Lavoisier, or Darwin when they themselves have never experienced a victory in analysis? How can they appreciate the amazing interweaving of the various sciences when each factual point is new and yet to be learned? The trees obscure the woods. How can we expect these students, coming to college for maturing, to enjoy and appreciate the maturest of disciplines at the very beginning of their maturation process? We cannot. Nor can we hope for happy memories about science from this unhappy adventure.

On the contrary, the only reasonable place for a broad physical science is toward the end of a student's college life, or at least after he has worked through such a course, as will be described presently. By then he is better able to work his way through a problem, is more resourceful and independent and more appreciative of ability and brilliant method. A science course at this later stage could accomplish, to a much greater degree, the desired ends of appreciation and understanding of the problems and methods of science.

A second objection to most physical science courses is the frequent lack of laboratory work, without which no person can understand a basic problem of science-measurement and its relation to theory. No amount of discussion at ease in the classroom can convey the fundamental contrariness of nature when man tries to unravel her secrets. A person who has never wrestled with experimental apparatus cannot know the many problems to be solved before a reproducible set of data can be published in a journal. How can he know of the times when the apparatus had to be rebuilt, when the meters had to be recalibrated, when glassware cracked and ruined a week's work, and so on? Or how can he know the tremendous satisfaction felt when data start to show consistency, proof of a theory starts to appear, and agreement with others is within the precision of the experiment? In other words, how can he know the basic motivation of most scientists? Granted that elementary laboratory work cannot provide all of the joys and frustrations, it can still give a much closer approximation than just talking about them.

The same type of objection holds, to a lesser degree, for the classroom demonstration lecture. Being a classroom attempt to show experimental results, it is a step in the right direction; but to be a good demonstration it must work the first time. So we come back to our difficulty again. The student vicariously performs an experiment which works perfectly the first time with almost no effort, mental or physical, on his part. Does he then know what science is? To my mind, no. Without the *proper kind* of experimental work, the physical science course must fail in its attempt to show the true nature of science.

Another poor feature of the physical science course is the widespread knowledge (or belief) of the nonscientific students that the course was created especially for them. Their general opinion is that such a course must be easier than the usual science course in order to enable them to "get through" their science requirement with the least possible pain. If this were not true, they opine, why did the faculty bother? The fact that "easiness" is not a typical characteristic in many cases still does not seem to change the student viewpoint. Obviously, this viewpoint is bad, particularly since it indicates another defect of the physical science course. This is the implication that people who are not scientists are essentially different from those who are. Thus, the provision of a physical science course for general education is automatically self-defeating. A common training is *not* provided for all. Instead, segregation of a kind almost as dangerous as that based on race is practiced at the very beginning of college education. Certainly that is no way to help stu-

dents realize the essential sameness of all lines of intellectual endeavor—the intensive use of the brain. To fulfill the desired objectives. the physical science course should be given to *all* students.

Finally, an objection to physical science courses can be based on teaching personnel. Any physical science course which tries to give an integrated picture of science covers several fields. But where can the average university, to say nothing of the college, find competent men trained in several fields and willing to spend most of their time teaching an elementary course? Such men are currently in the class of the neutrino. They have no rest mass. Thus, we have the sorry choice between a man of Ph.D. level teaching in a field where he has merely surface knowledge, or a man of much lower educational training and rank teaching the course because he is unable to teach at a higher level. In neither case can the student acquire many of the benefits claimed for the course.

THE TRADITIONAL APPROACH

Having pointed out some of the major defects of the usual physical science course, let us review briefly the defects of the typical general physics course. Its defects, of course, are some of the reasons for the growth of the physical science course replacing it. Usually the general physics course is broad but shallow. All branches of physics must be covered lest the student not be prepared for the next course in physics, or chemistry, or medical school. A very indigestible stew results, as many people have pointed out.

Usually the course is "professional" in nature. That is, the laboratory is more often a training school for the use of instruments than of physical principles. Analysis of a problem is highly mathematical; qualitative explanation is held to a minimum. The student is constantly reminded that the road ahead is long and difficult. What fun is there in studying physics, anyway?

Other defects are not necessarily characteristic of a general physics course. Still, they are defects which students associate with physics forever after. These are detailed, push-button type of laboratory directions and their boon companions, the Radio City Music Hall demonstration lecture. In both cases, the instructor is so enthralled with the perfection of his performance that he does not realize how little the student is learning. For the student cannot master physics without using his own mind to work things out. Nor, in the long run, will he enjoy it. Without mastery there is no joy of accomplishment, nothing but frustration and dislike.

Now, having tried to demolish the two current offerings of physics, what do we suggest as a replacement? The general outlines seem fairly clear from the objections previously raised. We must design a course with a limited objective which the majority can see and reach. We must unite all classes of students instead of separating and classifying them. We must unify and simplify our subject. We must have more student participation and hence enjoyment. All these are laudable objectives, but how are they to be reached in general physics? There is no one answer, human nature still existing, but we are reaching a form in our general physics course at St. Lawrence University which seems to produce the desired results. The general techniques are as follows.

First, there are no mass demonstration lectures. Each instructor holds a chatty combination of lecture, recitation, and demonstration with plenty of student questions. Demonstrations are spread through the three weekly classes; not compressed into one. The resultant smaller sections and ease of student-instructor intercourse seem much more effective than large lecture sections, even though one instructor may be a much better demonstrator than the others. By allowing thorough student questioning of each point or demonstration in turn, we can insure much greater student participation, interest, and learning.

This technique could have other benefits. In the universities, for instance, the standard of teaching done by graduate students could be raised tremendously, because of the elimination of the huge lecture demonstration. The resulting equal teaching responsibilities of graduate student and regular staff member should make the teaching graduate student more capable and effective. And simultaneously he would be given the chance to see for himself the joys of teaching and to decide whether or not such a job is for him.

Would not this method be a good start on the

problem of training Ph.D.'s in the teaching methods of their fields? We have found that the teaching abilities of our master's degree candidates develop rapidly in the stress of classroom give-and-take. At the same time, these candidates become much more humble about the extent and depth of their knowledge of physical principles. It seems clear that a person must have grasped the fundamental techniques of teaching if he can successfully instruct a class in beginning college physics!

The more intimate contact, necessary and possible between student and instructor, is another educational benefit. By personal acquaintance with a physicist, the student can learn what I believe to be a general truth. Physicists, or specialists in any field, are just as human as the students, but are unusually fond of their subject. This spark of enthusiasm conveyed in personal contact has kindled more fires in unsuspecting prospective physicists than any *blow* torch I know. This fact can be as true in a university as in a college.

Second, we have reduced the quantity of material studied to about two-thirds that usually covered in general physics. We omit, for example, discussions of specific gravity, density, Archimedes' principle, hydrostatics, surface tension, viscosity, friction, machines, thermoelectricity, photometry, some parts of geometrical optics, all topics which are useful but do not lend themselves readily to a continuity of approach. By this omission of topics, as Eric Rogers[1] has so cogently pointed out, the student is given enough time to master the topics considered. This mastery is very important if the student is to enjoy the course.

Also, by eliminating selected topics we can show a very definite continuity and coherence in physics. Instead of a jumble of topics studied primarily because they are "physics," we give a series of topics developing and using energy and its conservation as the skeletal support of the course. This device enables us to start with heat, a topic closely related to common experience, and develop a need for mechanics to explain the force concept required in a kinetic theory of heat. All through the rest of the course, energy in its acoustical, optical, electrical, and atomic manifestations serves as a familiar friend in a new costume. This concept is particularly useful in attacking the problems of the last fifty years of atomic and nuclear structure. By providing a clear, well-defined objective free of distracting underbrush, we can make physics more enjoyable and understandable for all students.

Third, we have deliberately abandoned a laboratory manual. The laboratory experiments are kept on an even front (sometimes we perforce slip to two different ones at once) and deal with some topic currently being studied in the classroom. Occasionally, an experiment supplements classroom work. In all cases, a brief statement of half a typewritten page outlines the objectives of the experiment and suggests, in a purposely vague way, the crucial measurements to be taken. From then on the student does the usual experiments in a manner as nearly like a research project as is possible in a two-and-one-half hour laboratory experiment. And it is a matter of observation that most college freshmen find it as difficult to decide methods and apparatus arrangements for a linear expansion experiment as a new graduate student would for a neutron velocity spectrometer.

For instance, in the above-mentioned linear expansion experiment, we use the well-known Cenco apparatus of a metal rod enclosed in a metal steam jacket with a micrometer screw for finding the change in length. After finding the initial reading of the micrometer, most students invariably record the few millimeters as the length of the rod (actually about 60 cm long). The value, to the students, of a five-minute discussion clearing up this point is tremendous—especially compared with merely recording said reading in the proper column of a printed data sheet. Of equal value is the amazement of the students when they find that the actual length of the rod may be measured with any old meter stick and still retain the same precision as the change in length measured with the micrometer.

No better illustration of the value of such informal, subtly directed laboratory work could be found than the remark of a student after about three such laboratory periods. He was repeating the course after failing the standard course previously. "You know, Prof.," he said, "this is the

[1] Earl J. McGrath, editor, *Science in General Education* (W. C. Brown Company, 1948).

first time I've really enjoyed physics." By finding physics to be fun, he will never go out and tell his children to avoid physics as the plague.

CLOSING THE BREACH

Obviously, this course is not intended to produce just professional physicists. It intends to teach students whose primary interests vary widely. Emphasis is on a coherent body of knowledge, methods, and attitudes which are typical of science. What better subject than physics to provide these things? Surely the nonscience student will benefit. And I believe the science student benefits equally by seeing that many intelligent students in other fields are capable of a good job in science but don't go on because the subject is not of major interest to them.

Actually, very few students entering college are positive of their life work. And the rest are given a chance in this course to see how real science appeals to them. If they decide science is not for them, they at least take away a pleasant memory and (we hope) increased knowledge. If they decide physics is for them, the little more they might have learned in a "professional" course will be unnoticable three years hence. Thus we feel that such a course as this, open to all, but a science instead of an "about science" course, is a necessary part of a college education.

Finally, it seems to me that the type of general physics course described would enable physicists to argue with great weight for a true physical science course for upper-class students. Such a course for seniors, more searching and analytical than it could ever be for freshmen, would be of great value. For then the broad view would be possible, based on knowledge already acquired and employing a mind as mature as college education can make it.

Many variations in emphasis and method should be possible. I give one as an example. Our philosophy department has recently taught a course in the philosophy of science to a group composed of students in the fields of biology, chemistry, physics, philosophy, and psychology. The course was of the seminar type, conducted by a philospher, with at least one scientist present. The analyses of basic scientific ideas that resulted were eye-opening to these science students. What similar results might not be possible with a mixture of nonscientists and scientists studying basic principles?

In conclusion, I propose the following. Whether or not we agree on the merit of the plan suggested above, we must stop having a purely defensive reaction to current educational theories on general education. Unless we can produce some educational ideas of our own representing an attack on the problem instead of a delaying action, physics in the liberal arts college will move closer to the attic. A vicious circle will start (or continue its present round) in which fewer high school teachers will be competently trained in physics and fewer people will care. Fewer entering college students will know there is such a subject, and fewer will elect it in college. General education will create the monster it is trying to destroy—a scientific class apart from the "common man" and an apt tool for unscrupulous schemers. To serve the best interests of a democratic society, we scientists must contribute as vigorously and prolifically to teaching as we now do to research.

Ninth Annual Pittsburgh Diffraction Conference

The Ninth Annual Pittsburgh Diffraction Conference will be held at Mellon Institute, Pittsburgh 13, Pennsylvania, on November 29 and 30, 1951. Sessions are tentatively being arranged on *Instrumentation and Methods* and *Metals and Alloys*. The Conference will also include a symposium of invited papers on the subject of *Diffraction and Thermomagnetic Studies of Reactions of Metastable Carbides of Iron.*

For further information, and for a copy of the preliminary program when available, write to Mr. C. W. Cline, Aluminum Research Laboratories, P. O. Box 772, New Kensington, Pennsylvania.

NOTES AND DISCUSSION

A Short Radius Optical Lever for Use with Young's Modulus Apparatus

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IN connection with a recent change in the Young's modulus apparatus used in our general physics laboratory, we had need of a short radius optical lever. No suitable one being available for purchase, we made our own as described below and shown in Fig. 1. It has a radius of one centimeter and can be used with very small wire or with wire a millimeter or more in diameter.

The optical lever is a wheel 2 cm in diameter made from a $\frac{5}{8}$-in. o.d. ball bearing with a brass rim, press fit, on the outer race turned to a 2-cm diameter and mounted on the end of a $\frac{1}{2}$-in. brass rod. The mirror, from a mechanic's inspection mirror, is attached to a curved bronze spring which fits tightly part way around the optical lever wheel.

A $\frac{3}{16}$-in. brass rod, fastened in a hole in the side of the half-inch rod, is bent at right angles as shown, and has attached to it a 20-gauge bronze spring which supports a small wheel that is held firmly against the optical lever wheel.

In use, this optical lever is supported on the Young's modulus apparatus frame by means of a cross clamp. The wire passes between the optical lever wheel and the small wheel on the spring and is held against the optical lever wheel with sufficient force so that there is no slipping. The wire may be straight, or bent slightly, where it passes between the wheels.

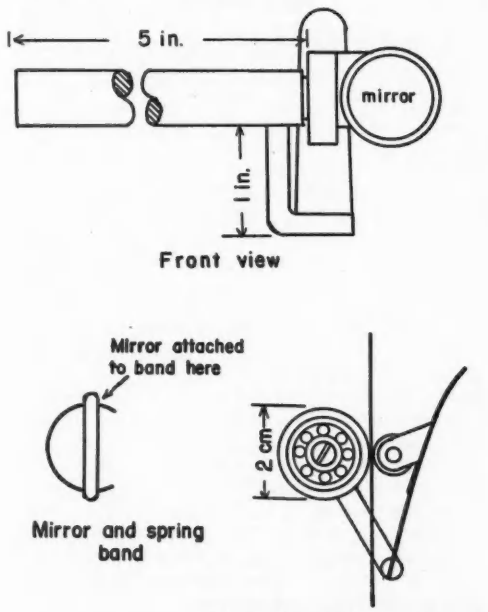

FIG. 1. Short radius optical lever for use with Young's modulus apparatus. The mirror is fastened at one place only to a circular spring band which fits snugly over the rim of the 2-cm pulley that forms the lever proper.

This arrangement gives us an optical lever of *short* and *constant* radius. With a telescope and scale about 180 cm from the mirror a 10-kg weight on a spring-brass wire of about one-millimeter diameter and one-meter length will give a deflection of about 40 cm.

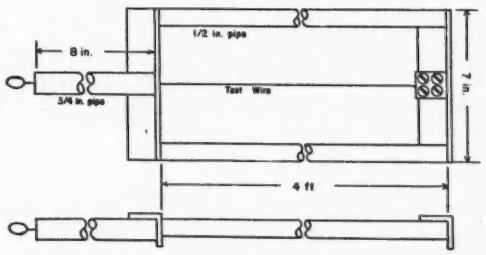

FIG. 2. Simplified form of the usual Young's modulus apparatus. It is clamped by the lower 8-in. length of pipe to the edge of a laboratory table.

A fiber wheel made by turning down a one-inch ball bearing pulley, such as is used with airplane control cables, has been tried. It seems to be somewhat easier to work with, and we expect to replace the brass-rimmed wheels with fiber wheels when time permits.

A brief description of the Young's modulus apparatus with which the optical lever is used might be of interest. It is of the usual rectangular frame type, Fig. 2, the top and bottom members being of $\frac{3}{16}\times 1\frac{1}{2}\times 1\frac{1}{2}$-in. angle iron and the side columns of $\frac{1}{2}$-in. water pipe, all welded together. Instead of supporting this frame on a heavy tripod base, as is commonly done with similar apparatus, a piece of $\frac{3}{4}$-in. pipe about 8 in. long is welded to the bottom angle iron and the whole apparatus fastened to the end of a laboratory table with a heavy table clamp.

The wire is attached to the upper angle iron by a suitable clamp, passes through a small hole in the lower angle iron and down through the $\frac{3}{4}$-in. supporting pipe, ending in a loop from which the weight holder is hung.

This flat equipment is easy to store by simply hanging it against the wall.

A Simple Relation for an Achromatic Telescope Objective

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THE author wanted to convert an achromatic lens of 8-in. focus and nearly 2-in. diameter into a long-focus telescope objective by regrinding and polishing the lenses. The original lens consisted of a cemented combination of an equi-convex crown and a plano-concave flint, the concave registering with the convex. This is one of the

most common types of small telescope objectives. While making theoretical computations for the achromatism of the new long-focus combination, a simple relation for this kind of lens was discovered. I found firstly that, assuming the original lens to be achromatic, it will remain achromatic even when it is ground into a long focus, provided the new lens is of the same type, i.e., equi-convex crown and plano-concave flint. I found secondly that the condition for achromatism of such a lens is given by the simple relation that the mean dispersion for the flint must be twice the mean dispersion for the crown lens.

The usual procedure for working out the condition for achromatism is to start with the formula

$$(\omega/f)+(\omega'/f')=0,$$

where ω stands for dispersive power and f and f' are the focal lengths of the components. Or, knowing that one lens is convex and the other concave, and neglecting the sign,

$$\omega/f=\omega'/f', \quad \text{or} \quad f'/f=\omega'/\omega.$$

Glass manufacturers in their data give, not the dispersive power but its reciprocal, the Abbe number ν. Thus the relation becomes

$$f'/f=\nu/\nu'.$$

This condition together with the relation

$$1/F=(1/f)+1/f'$$

for the focal length F of the combination is used to work out what the radii of curvature of the four surfaces should be. For a combination of the type mentioned above the ν values are so chosen as to make the radius of curvature of the fourth surface so large that the surface may as well be plane. Suitable glasses for such combination are recommended by manufacturers.

It is shown below that the problem can be approached from another angle. One need not refer to the ν-values at all. The glasses can be more conveniently selected from mean dispersion values. Thus, supposing an achromatic combination of the above description is to be built, what properties must the glasses have?

We have the relation

$$f'/f=\nu/\nu'.$$

Also for the equi-convex crown

$$1/f=2(n_d-1)/R,$$

and for the plano-concave flint of the same radius

$$1/f'=(n_d'-1)/R,$$

where R is the radius of curvature of each of the three equal surfaces, and n_d the mean refractive index. From these three relations we get

$$f'/f=2(n_d-1)/(n_d'-1)=\nu/\nu'.$$

Or substituting the usual values for ν and ν',

$$2(n_d-1)/(n_d'-1)=(n_d-1)\Delta'/(n_d'-1)\Delta.$$

Thus

$$\Delta'=2\Delta,$$

where Δ stands for the mean dispersion, i.e., for (n_f-n_c), where n_f is the refractive index for blue and n_c is for red.

The mean dispersion is also a quantity given in the manufacturer's tables. One can therefore select a pair of optical glasses which satisfy the above condition. The Abbe number is not required at all.

Inasmuch as the above relation will hold whatever the value of R, the pair that satisfies this condition will make an achromatic combination of any desired focal length so long as the type of combination remains the same. It is also found that pairs recommended by manufacturers for this type of combination satisfy my condition very closely. (It must be mentioned that this formula is implicit in and can be derived from Conrady's formula[1] $C_a/C_b=dN_b/dN_a$, C standing for the curvature of the lens and dN for the mean dispersion. But Conrady derives it from more general considerations and does not apply it to the above case or to any actual case.)

It must be pointed out that in any actual case the value of R for the desired focal length will have to be worked out from usual formulas, but this will also not require Δ or ν. Thus, the focal length of the combination in this case will be given by

$$\begin{aligned}1/[F=2(n_d-1)/R]-(n_d'-1)/R\\ =\{2(n_d-1)-(n_d'-1)\}/R\\ =(2n_d-n_d'-1)/R,\end{aligned}$$

so that from the known values of n_d and n_d' the radius of curvature for any desired focal length can be found.

[1] Conrady, *Applied Optics and Optical Design* (Oxford University Press, 1929, printed again in U. S. A., 1943), p. 148.

A Vapor Pressure-Temperature Apparatus

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AND
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A SIMPLE vapor pressure *vs* temperature apparatus may be easily constructed from readily available galvanized pipe fittings. This apparatus, requiring less than 100 cm³ of liquid, may be used at a greater range of pressure (and temperature) than equipment usually described in laboratory manuals.

The apparatus, as shown in Fig. 1, is constructed largely from 1½-in. pipe fittings using a nondrying pipe thread lubricant. A vise and 24-in. Stillson wrench can be used in tightening this pipe, without any fear of stripping the threads. Where access to the interior is needed, it is desirable to reduce to 1 in. so that the smaller part may be more readily disconnected.

The thermometer well is made of a 1-ft length of ½-in. copper tubing, capped at the bottom. This well extends out through the bushing at the top of the water chamber and should be brazed into that bushing with silver solder. (Caution: Do not allow any of this silver solder to get on the pipe threads. It is too difficult to remove.)

The cooling coil in the condensing chamber, made by coiling 2 ft of $\frac{3}{16}$-in. copper tubing, should extend from the pipe plug at the top to a point near the elbow at the bottom. This coil makes entry and exit through the plug at the top. Silver solder should also be used on these joints.

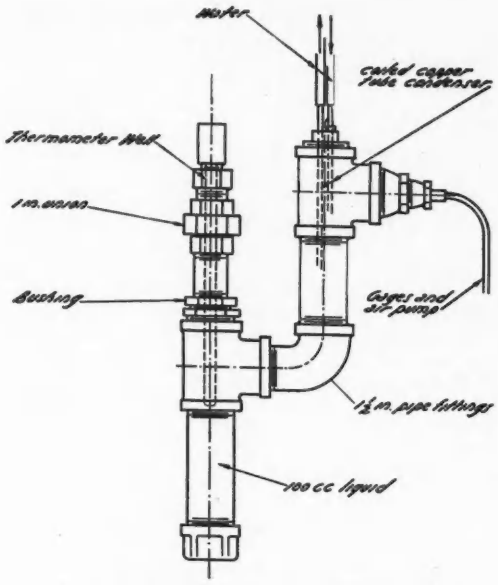

FIG. 1. A vapor pressure-temperature apparatus made of standard galvanized pipe fittings.

The pressure-equalizing tank and gauges are a matter of choice. A large number of oxygen tanks rated at 400 lb/in.², 6 liters' capacity, and having openings at both ends seem to be available on the surplus market. Since schools usually do not possess mercury columns registering much over one atmosphere, a Bourdon gauge will no doubt be used for higher pressures. Pressure may be supplied from compressed air systems, or by means of an automobile tire pump. The latter, for a student, is an almost self-regulating safety device, for the maximum pressure which can be developed is about 90 lb/in.² and many persons are willing to stop at a lower figure. The pipe specified above, however, normally operates at many times that pressure. A water aspirator pump is sufficient for vacuum.

Experience indicates that if a mercury column is used, an equalizing tank should be placed between the apparatus and the mercury column, so that pressure is transmitted through the tank. Otherwise, at low pressures, excessive pressure variations, resulting from violent boiling, may result in large oscillations of the mercury column.

The use of the small diameter ($\frac{3}{16}$-in.) copper tubing or hose between the two pressure vessels does have one important advantage in that if accidently broken by a student, the release of the air will be noisy but rather slow. Copper tubing tends to harden with use, but it can be softened by heating to a red heat.

The vapor pressure apparatus described above is simple to operate. A single Bunsen burner, which *must* be operated at a considerably reduced flame length during low pressure measurements, will supply adequate heat at high pressures. The thermometer follows pressure variations quickly at high pressures, but at low pressure the equilibrium temperature is reached more slowly. Tests made in draft-free rooms do not seem to indicate the necessity of heat insulation.

The laboratory problem may be selected to suit the instructor's desire. If a dead weight gauge tester or a high pressure mercury column are not available, the problem might be directed towards testing the gauge by means of steam tables. More advanced work, no doubt, should include plotting vapor pressure *vs* reciprocal of the absolute temperature on semilog paper and the determination of the constants in the familiar equation, $\log p = B + A/T$. The current experimental determination of vapor pressure data is now resulting in a surprisingly large number of papers. Certainly, the instructor should not only introduce the student to tables in handbooks, but also to the critical compilation of data by Stull.[1]

[1] D. Stull, Ind. Eng. Chem. **39**, 517, 540 (1947); **40**, 496 (1948).

Some Criteria for Choosing Laboratory Experiments

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I WONDER if the committee[1] which is collecting descriptions of laboratory experiments should not also be asking why these experiments are recommended and used. I have no doubt that Professor Lloyd W. Taylor, in whose honor these experiments are being collected, would have an interest in the educational uses to which they are to be put.

In the April, 1951, issue there was a description by Julius H. Taylor[2] of *An Apparatus for Measuring the Acceleration Due to Gravity*. I would like to ask why anyone should do that experiment when there are already many ways of determining the same constant—and then try to give some possible answers; there are undoubtedly others. I ask the author's pardon for concentrating my attention on his suggested experiment; it was the one that started this reaction.

1. He offers one reason for doing this experiment; the apparatus is simple and easy to operate. It is not as simple and as easy to operate as a simple pendulum and it does not give as good results for the acceleration of gravity. Nevertheless, the reason is a valid one in general; we may choose to do a certain experiment at a certain time because of its simplicity and ease of operation.

2. We might choose a certain way of measuring the constant because it gives better values than any other method available. That does not apply in this case.

3. We might use an experiment in an educational laboratory because it offers an effective vehicle for teaching some aspects of physics that we think are important. For example, the Behr method and other free-fall methods offer opportunities for teaching, early in a course, things having to do with curve plotting, successive differences, deviations, and other statitistical devices. Because of that we might use it despite the fact that it does not give particularly good results for the acceleration of gravity, and

in spite of the fact that its complicated spark system is not understood by the student. The value of the acceleration of gravity is practically a by-product; the main product is what the student learns. In the same way, Professor Taylor's experiment can be used to teach something about centripetal acceleration.

4. We might choose to do it only because it is a delightfully simple and interesting experiment which we can hope will rouse the student to some enthusiasm for doing it no matter how good the results as measured in terms of percent error. This I think is the best justification for the experiment suggested by Professor Taylor.

[1] Am. J. Phys. **19**, 145 (1951).
[2] J. H. Taylor, Am. J. Phys. **19**, 245 (1951).

Effect of Aeration on the Viscosity of Water*

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SOME measurements have been made of the kinematic viscosity of aerated distilled water indicating a definite increase of viscosity with aeration. A search of the literature has failed to reveal any experimental evidence of this effect which is of importance in hydrodynamics investigations.

In the present work distilled water was de-aerated by boiling and evacuating. Samples were aerated to various degrees by cascading them through aeration tubes which were described earlier.[1] These tubes were so constructed as to allow rising compressed air to mix with the water falling in the tube at screen-baffles. The kinematic viscosity of the samples was obtained by measuring repeatedly the efflux time of 3.5 ml through a capillary-tube-type viscometer held in a water bath. Aeration was determined by measuring the dissolved oxygen content of the sample by Winkler's method.[2] Samples appeared bubble free, but this method does not differentiate between dissolved oxygen and that in the form of minute bubble nuclei. However, it is felt that aeration generally is not necessarily all dissolved air.

At low oxygen concentrations the samples had values for viscosity which were in good agreement with those in tables. At 4° the kinematic viscosity was increased 0.5 percent for each ppm (part per million) of oxygen up to 11 ppm. At 24°C the effect was increased for values above five ppm to 6 percent elevation of viscosity per ppm of oxygen. This may explain the difficulty one has in checking tabular values of viscosity by various laboratory methods at room temperature, since distilled water standing open to the air usually contains about eight ppm of oxygen. Also, in utilizing tabular values for viscosity in hydrodynamics calculations (such as Reynolds' number), an error caused by this aeration effect may be introduced.

Of course, oxygen is not the only gas present in aerated water, but it can be safely assumed that the amount of oxygen indicates the amount of other gases present at that temperature according to their respective solubilities. As an approximation based on handbook values, the aeration may be considered to be one-fifth oxygen.

These preliminary measurements are being repeated in greater detail with apparatus of higher precision. The study is being extended to higher temperatures and the effect of pressure is being included. A later communication will present these results.

* This work was sponsored by the U. S. Navy Department under Contract NOrd 7958.
[1] C. H. Tindal and D. C. Whitmarsh, Am. J. Phys. **16**, 300–303 (1948).
[2] T. G. Thompson and R. J. Robinson, J. Marine Research **2**, 1–8 (1939).

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LETTERS TO THE EDITOR

The Propagation of Errors in the Elementary Course

A RECENT article[1] states two rules for the manipulation of numbers having regions of uncertainty. For a beginning student it should be made clear (1) that these rules apply only to the maximum region of uncertainty of the result and (2) that the maximum region of uncertainty has limited usefulness compared with the more widely used *most probable* or *root-mean-square* region of uncertainty.

Calculation of the perimeter, P (area, S), gives four possible values, only two of which, the maximum and minimum, are given by Eq. (1) [Eq. (2)] of the article.[1] However, since the signs to be used with a and b are independent and unknown, all four values of P (of S) are equally probable. Theory [2] shows that in this case the most probable region of uncertainty is calculated according to a root-sum-square law. Since for most cases it is more desirable to know the most probable region of uncertainty, rather than the maximum, and since the former is the one with which students will come in contact more and more in their later work, it is advisable to start them on the two rules stated by Birge[3] rather than on those given by Fleisher and Olsen.

The rectangle example, chosen by Fleisher and Olsen, not only shows the application of Birge's rules[3] for addition (subtraction) and multiplication (division), but may also be used to indicate the difference between independent and dependent regions of uncertainty. Since Birge's rules apply to independent regions of uncertainty, the region of uncertainty of the perimeter would be found from $P=2(A_0+B_0)$, rather than from $P=A_0+A_0+B_0+B_0$. In this connection see remarks by Birge.[4]

For beginning students Birge's rules will take care of most calculations. As soon as possible, however, they should be given the basic equation[5] for the propagation of errors.

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[1] H. Fleisher and L. O. Olsen, Am. J. Phys. **18**, 51–52 (1950).
[2] See any standard text on the theory of errors; also, R. T. Birge, Am. J. Phys. **7**, 351–357 (1939).
[3] See R. T. Birge, reference 2, rules (a) and (e), pp. 352–353.
[4] See reference 2, p. 354.
[5] See reference 2, Eq. (4).

An Extension of the Falling Chain Problem

IN earlier notes[1,2] I considered the weight of a falling chain. A chain of length L and total mass M is held vertically *by one end* and allowed to fall. The maximum reading of the scale (balance) upon which it accumulates is shown analytically to be $3M$. When the balance is a spring scale, the maximum reading experimentally is $3M$, as the analysis requires, but with a platform beam balance the maximum reading experimentally is $2M$. This dilemma is not readily resolved. In pursuing the investigation I was led to consider a modification of this falling chain problem which I wish here to report. It is with no little astonishment that I discover still another dilemma.

The chain in this case is held vertically *at its two ends*, the loop being a U of small width. The ends are at the same horizontal level. One of the ends is let go. A number of questions may be asked: (1) What is the tension at the bight as the chain goes over from the falling side to the fixed side? (2) What is the maximum pull on the support of the fixed side? (3) What maximum reading will a spring scale show to which the stationary end is fixed?

It is an interesting exercise for a class in analytical mechanics to discover the following:

1. The upward force at the bight (on the fixed side, obviously) is at any time $\frac{1}{2}mv^2$, where m is the linear density of the chain and v the velocity acquired by each element under free fall.
2. At the support on the fixed side the pull at any time t is $\frac{1}{2}Mg+\frac{3}{4}mv^2$.
3. At the instant of final transfer (when the last of the falling chain passes over to the fixed side) the pull on the support becomes $2Mg$. At this instant the pull at the bight is Mg. This goes to zero as the mass-transfer is completed and the pull on the support decays to Mg.
4. The initial potential energy of the falling side is $\frac{1}{4}MgL$. This is exhausted in the fall. It should be identical with the kinetic energy appearing at the bight. That this is so can be shown.

Of the questions put, the first two are answered. Can we answer the third from the analysis? *Peculiarly, the maximum reading of a spring scale to which the stationary end is fixed is not $2Mg$. What is it and where lies the dilemma?*

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[1] J. S. Miller, Math. Mag. 21, 159 (1948).
[2] J. S. Miller, Am. J. Phys. 19, 63 (1951).

Falling Chains

TWO letters[1] by Mr. J. S. Miller on falling chains raise interesting points. The letters are based on Exercises 6 and 5 on p. 149 of Lamb's *Dynamics*.[2] Mr. Miller has carried the problems farther than Lamb's exercises and has apparently also performed the experiments. In one case, he finds himself in a dilemma which I think I have solved. I also have carried out the experiments with the results noted below.

1. A freely falling chain of mass M at the instant of complete accumulation should exert a force on the platform equal to $3\ Mg$. To test this by experiment, No. 14 jack chain of length 315 cm and of mass 245 g (=8.6 oz) was used. I attached one end to a fine string, and flung the other end of the string over a horizontal pipe near the ceiling of my room. I then hauled up the chain until its lower end was just clear of the pan of a dial-reading spring balance (like a kitchen spring balance). It read up to 10 pounds in ounces. Releasing the string, the chain fell and the pointer of the balance sprang up to 26 ounces (variations from 24–28). In doing the experiment it is hard to see just how far the pointer moves, but by putting one's finger on the dial, it is possible by the sense of touch to record the maximum reading of the pointer. The average reading, 26 ounces, is very nearly three times 8.6 ounces so this confirms the theory. The pan descends only a very little way.

Mr. Miller says, however, that a platform balance reads only $2\ Mg$ when the experiment is actually performed. I replaced the spring balance by a two-pan lever balance[3] and repeated the experiment. To distinguish between the $2\ Mg$ and the $3\ Mg$, I loaded one pan in turn with 490 and 735 g, held the pans level with my hand and noticed which way the "kick" went when the chain fell on the unloaded pan. The 735 g was much nearer the final force. Others confirmed with me that the final force was certainly greater than the $2\ Mg$ and approached nearly the $3\ Mg$ of theory.

2. In his second letter Mr. Miller poses a problem that is a little harder, and to make my remarks clear, I must give the analysis. The chain, of length L, has a mass m per unit length. Assume the free part of the chain falls with acceleration g. When the free end of the chain has fallen a distance x, the upturned part of the chain is descending with velocity v, given by $v^2=2gx$; and in a short element of time dt the free chain descends dx, and $dx/2$ of chain goes around the bight and becomes stationary.

The decrease of momentum in time dt is $mvdx/2$. Therefore, the force in the chain on the fixed side of the bight is $(1/dt)(mvdx/2)=\frac{1}{2}mv^2$. It is hard to check this by experiment. However, just before the fall is completed, $v^2=2gL$ and the force on the spring balance should be mgL owing to the (now) stationary load plus $\frac{1}{2}m(2gL)$, bringing the total force to $2\ mgL$, or $2\ Mg$. Mr. Miller says the maximum reading of a spring-balance scale to which the stationary end is fixed is not $2\ Mg$. What is it and where lies the dilemma?

Before answering Mr. Miller, let us consider the chain in some intermediate position x at time t. The force F on the support of the fixed end may be considered made up of three parts and may be written $F=$wt of length $L/2$ of chain (this is always present)+wt of length $x/2$ of chain (which has come around the bight)+the pull at the bight. That is, $F=\frac{1}{2}mgL+\frac{1}{2}mgx+\frac{1}{2}mv^2=\frac{1}{2}mgL+\frac{3}{4}mv^2$, and the maximum force is $2\ Mg$, as before.

I repeated the experiment using 208 cm of No. 14 jack chain, of total mass 155 gm, determined by the tubular spring balance used. This balance read to 500 gm by 10's,

stretching 9 cm for the maximum load. When the chain was doubled up preparatory to the drop, the reading fell to 78 gm; and when the free end was released and the half chain fell, the balance reading jumped up, at first I thought, to 470 gm but later to 500 gm. (Again I used my finger to see how far the index came down). This balance may not give the maximum reading, so I replaced it by a larger balance reading up to 4 lb by ounces (it stretches 3 cm per lb) and the index now recorded a maximum reading of 1 lb 4 oz or 567 gm, which is nearly 4 Mg. Now why is this?

There is an old experiment of dropping a load M from *just* above the scale pan of a spiral spring balance (or even on the pan supported by a wire in a Young's modulus experiment) upon the scale pan. G. F. C. Searle[4] describes it. The reading momentarily increases to that which would be given by a statical load of $2M$. Searle proves it. So possibly Miller's dilemma is due to the give of the spring balance.

To check Searle's remark, I held a 500-gm weight *just* above the pan of the dial balance used in the first experiment and suddenly released it. The index jumped to 2 lb 3 oz and eventually, after several vibrations, dropped back to about 1 lb 2 oz, the static load.

I also checked it with the whole length of the chain hanging from the 500-gm spring balance. Holding the topmost link just free of the spring's hook, the reading was, of course, zero. Suddenly releasing it, the pointer jumped to 320 gm, i.e., to twice the statical load. With the 4-lb spring balance (statical load nearly 6 oz) the index jumped to 12 oz. Possibly the chance of getting an accurate 2-Mg reading depends on the mass of the movable part of the balance and the distance it extends. My readings are approximate.

Let us come back to Miller's experiment and the theory where F is given as $F=\frac{1}{2}mgL+\frac{1}{2}mgx+\frac{1}{2}mV^2$. Since the last two terms represent forces which are suddenly applied, we might expect the spring balance to read $F'=\frac{1}{2}mgL+2(\frac{1}{2}mgx+\frac{1}{2}mV^2)$, and its maximum reading to be, therefore, $F'=\frac{1}{2}Mg+3\ Mg=3\frac{1}{2}\ Mg$. It is not easy to distinguish between $3\frac{1}{2}\ Mg$ and $4\ Mg$.

These chain experiments, and there are others given in Lamb, Love, and Routh, afford pretty exercises in mechanics. Some of our younger men here got very interested in my experiment and tackled the theory. One thought that the assumption of an acceleration g for the falling chain of Miller's second letter might be too rash and likened the motion of the falling portion of the chain to the motion of a falling rain drop, which is continually evaporating. Another tackled the problem by dealing with only the falling portion of the chain. Using the expression, Change of Momentum = Force $\times dt$, he wrote $\frac{1}{2}(L-x)mdv-m(dx/2)v=\frac{1}{2}gm(L-x)dt$, whence $(dv^2/dx)-[2/(L-x)]v^2=2g$. The solution of this equation is $v^2=\frac{2}{3}g\{[L^3-(L-x)^3]/(L-x)^2\}$, which gives $v=0$ when $x=0$, $v^2=(7/3)gL$ when $x=L/2$, and $v=\infty$ when $x=L$. So there must be something wrong in basic assumptions.

I remember J. J. Thomson saying that when an electron was stopped by a target, a wave moved out along its lines of force like a flick in a whip. Now we know that this flick gives a very intense motion at the very end of the whip. Possibly the last piece of chain as it rounds the bight behaves like the flick.

In reference 4 Mr. Miller deals with the mechanical energy, but I think it is a bit dubious to employ the conservation of energy in impulse problems. Taking a datum at level L before the top of the chain, the total potential energy at the start is $\frac{1}{4}\ MgL$, and in the position x the total potential and kinetic energy is $\frac{1}{4}\ MgL-\frac{1}{4}\ Mgx^2$. So that when x has become L the total mechanical energy is $\frac{1}{2}\ MgL$, which is the potential energy of the chain at final rest. The difference has been frittered away in heat.

Very interesting, Mr. Miller. So glad you called our attention to falling chains.

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[1] J. S. Miller, Am. J. Phys. **19**, 63 (1951); Am. J. Phys. **19**, 383 (1951).
[2] Horace Lamb, *Dynamics* (Cambridge University Press, London, 1947), second edition.
[3] Central Scientific Company, Trip Scale No. 3470, Catalog J 141.
[4] G. F. C. Searle, *Experimental Elasticity* (Cambridge University Press, London, 1908), p. 82.

Apparatus Wanted and for Sale

MANY college physics departments have duplicate or discarded equipment that could be used elsewhere. Could not the *American Journal of Physics* serve us by making known such items?

At the University of Pittsburgh, we can spare the following: 1 Erpi, 16-mm sound film, perfect condition, at half price, $22.00; 8 Linear expansion devices, micrometer type, Cenco No. 77,410, half price, each $15.00.

We urgently need: 1 to 3 Millikan nonelectrical, falling tuning fork acceleration-of-gravity devices.

O. BLACKWOOD

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Practical Laboratory Experience

THE department of physics at the University of Portland offers a course in "Laboratory Practice" which might be of interest to other schools. It was introduced about four years ago with the dual purpose of instruction and economy. During the years of peak enrollment, the general physics laboratory sections were beyond the capacity of the regular staff of instructors. Without the help of graduate assistants, we were at a loss to find the necessary guidance for the elementary experiments. At first we hired some of the senior physics majors at the current rate for student help. Naturally we selected the best in the class. But their reaction was so informative that we decided to make the work compulsory for all majors. As an added incentive we allowed one semester hour of credit with a maximum of two credits. At the present time, there is no remuneration except the credit and the experience.

Those enrolled in the course meet once a week for a briefing on their duties. This regular class period is held at the end of the week so that while the instructions are

fresh, the students can arrange the laboratory rooms for the coming set of experiments. We ordinarily run four set-ups of three different experiments at the same time, and thus change equipment every three weeks. Each laboratory assistant is given charge of a different set-up (four tables) each week. Thus during the year he covers all the experiments in the general physics course. He is required to gather all necessary equipment from the stockroom, see that it is in working order, arrange the required tables with auxiliary materials, and then run the experiment himself. A completed data sheet with results is demanded, so that the complete details of the work will be clear in the mind of the assistant.

The first hour of the laboratory period is devoted to direction and quiz. The assistant is in charge of the students assigned to a particular experiment for that day. He explains, where necessary, the procedures, noting changes from the original printed directions, and stresses the particular dangers or difficulties which may be encountered. For the rest of the hour he is expected to answer the questions of the students. Of course, the instructor supervises all this without taking part except where necessary. He also "listens in" on the direction of the experiments, and devotes particular attention to those who are able to do supplementary work, and to those who are especially slow.

At the end of the period the assistants check the data sheets and the results, then grade the students for the day's work. The laboratory grades thus become quite objective, although our experience has been that the student assistants tend to grade all alike and generally *lower* than the instructor normally grades. This has been true even of those who are very critical of their own grades. We are convinced that this grading practice is good pedagogical psychology.

After the dismissal of the class, the assistants make recommendations for changes in the experiments. Some of the outstanding seniors have made a few valuable suggestions and at times have designed and made more efficient equipment.

Those students who have completed the course have consistently told us that they learned more from that course than from any other single course in their program. This should be expected because it contains all the elements for learning. Naturally some, particularly at the beginning, look on the class as just another burden. This number decreases as the year progresses and after one or two years the word gets around that there is a great advantage in the experience.

The benefit of economy is obvious. Smaller schools that do not have the help of graduate students must sacrifice either the salary of extra help in the laboratory, or the number of people in direct contact with the students performing the elementary experiments. An understaffed laboratory tends to destroy interest because of delays in what is too often a tedious duty. Without increasing our cost, we have added ample help, and at the same time a youthful zeal that supplements the experience of the staff. The instructors have more time to devote to the exceptional student and the deficient.

One difficulty encountered is the tendency of the youthful assistants to give an answer to a question when they are not sure. We have found it necessary to emphasize the fallacy of this procedure and to explain that there is no disgrace in admitting that one is not ready to given an answer at the moment. The instructor must be on hand at all times to catch errors.

We heartily recommend the adoption of this course to all schools which have physics majors.

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Erratum: Motion Relative to the Surface of the Rotating Earth

[Am. J. Phys. 19, 53 (1951)]

AN error occurred in the derivation of Eq. (3). The expression for the angle between the radius vector and the normal of an ellipse should be

$$\tan\beta=(a^2-b^2)\sin\phi\cos\phi/(a^2\sin^2\phi+b^2\cos^2\phi).$$

Since a and b are very nearly alike, and since β is a very small angle, this reduces to the approximation:

$$\beta=2(a-b)\sin\phi\cos\phi/r. \qquad (3)$$

Equation (6) then becomes

$$\mathbf{G}=-[(32.2577-0.0587\cos^2\phi)\mathbf{r}_0+0.1056\sin\phi\cos\phi\boldsymbol{\Phi}].$$

The term, $0.0587\cos^2\phi\mathbf{r}_0$, is larger, and the term, $0.1056\sin\phi\cos\phi\boldsymbol{\Phi}$ is smaller, than the quantity computed from newtonian potential theory for a homogeneous oblate spheroid.

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Snell's Law Equivalent to the Conservation of Tangential Momentum

HISTORICALLY, the association of Snell's law and momentum conservation (the momentum, as for a material particle, being considered directly proportional to the light velocity **V**) was apparently exploded by Foucault's demonstration that the velocity of light in water is less than in vacuum. With the proper definition of momentum for light (inversely proportional to **V**), Snell's law and the conservation of the tangential momentum are found to be equivalent.

According to the de Broglie formula, the momentum of a photon is

$$\mathbf{P}=h\omega/\mathbf{V}=(h\omega/\mathbf{V})\cdot(\mathbf{V}/\mathbf{V})=h\omega\mathbf{V}/V^2, \qquad (1)$$

where **P** is the momentum, ω the frequency of the light, and h is is Planck's constant.

In Fig. 1, which represents the refraction of light from vacuum to glass, the interval **V** drawn parallel to the propagation direction is proportional to the light velocity in vacuum. The interval V_x is the component of **V** which is parallel to the glass surface. Likewise, an interval $\mathbf{V}_g$ proportional to the light velocity in the glass and its tangential component V_{xg} are drawn. The components of

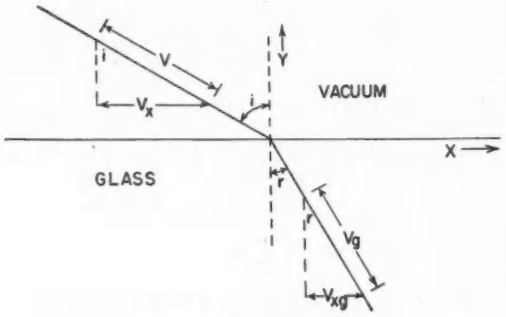

FIG. 1. Light refracted by plane glass surface.

momentum P_x and P_{xg}, parallel to the interface in vacuum and glass, respectively, are therefore given by

$$P_x/h\omega = V_x/V^2 = (\sin i)/V, \tag{2}$$

and

$$P_{xg}/h\omega = V_{xg}/V_g^2 = (\sin r)/V_g. \tag{3}$$

Assuming that the refracting medium exerts no "force" parallel to the interface on the incident photon, the x-component of the momentum is conserved; that is,

$$P_x = P_{xg}. \tag{4}$$

Multiplying this by $(1/h\omega)$, one finds

$$(\sin i)/V = (\sin r)/V_g. \tag{5}$$

By definition, the refractive index n_g of the glass is given by

$$n_g = V/V_g. \tag{6}$$

When combined, these last two equations express Snell's law in its familiar form:

$$\sin i = n_g \sin r. \tag{7}$$

Conversely, from Snell's law one may prove that the tangential momentum of a refracted photon is conserved. The equality of the incidence and reflection angles demonstrates the conservation of the tangential momentum of a reflected photon.

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Selective Heating Effect in the Radiometer

AN interesting observation in what appears to be selective heating by high frequency electromagnetic energy has been observed with radiometers. A radiometer with aluminum vanes, shiny on one side, dull black on the other, in the presence of sunlight revolves in such a manner that the dull black side recedes from the radiant energy. Likewise, radiometers made with mica vanes, dull black on one side and shiny on the other, rotate in the presence of radiant energy (sunlight, incandescent lamp, bathroom heater, etc.) in such a manner that the dull black side recedes. When the two radiometers are placed in the presence of microwaves (radar) of high frequency (wavelength approximately 11¼ cm), the radiometer with vanes made of aluminum reverses its direction and moves with the shiny side receding, whereas the mica blades of the other radiometer continue to move in the same direction, that is, with the dull black sides receding. In short, the vanes of the two radiometers revolve in the same direction in the presence of sunlight, artificial light, etc., but the radiometer with aluminum vanes revolves in the reverse direction in the presence of microwave energy.

The following reason is given for the vanes of the radiometer acting as they do.

Each vane has one side polished and the other blackened, and they are free to rotate in a partial vacuum. When radiant energy from the sun or other intense source falls upon the polished face, it is reflected; and when the radiation falls upon a blackened face, it is absorbed and that face becomes hot. As the air molecules in the bulb dart about, some will impinge upon the vanes; those that strike the hot blackened face will rebound with increased velocities and cause the pivoted element of four vanes to rotate.

When the two radiometers are in the presence of the sun or other conventional radiant energy sources, they rotate with the blackened side receding. On the other hand, when exposed to microwaves, the vanes made of aluminum absorb the microwave energy and become hot, whereas the mica vanes do not absorb as much energy. Now the aluminum vanes are giving off heat from within them. Thus, the blackened side, being a better radiator, is cooler and the shiny side, being a poor radiator, is warmer.

The impringing and departing molecules in a "vacuum" impart a force on the shiny side of the aluminum vanes which causes them to rotate with the polished face receding.

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Physics via Television

THE week of February 25, 1951, marked the ninth for the WFIL-TV (Philadelphia) "University of the Air," a public service program of adult education, presented five days of the week for fifty minutes each day. Although this was the first extensive use of the medium of television for an educational program series in the Philadelphia area, it received widespread commendation. Professors from some fifteen colleges and universities in this area gave their services in the presentation of courses designed to bring the university into the home.

It was the writer's privilege to present on the University of the Air an eleven-lecture series on the subject "Nuclear Physics for the Layman." The subjects discussed included matter and electricity, radiation, radioactivity, isotopes, nuclear transformations, tracer elements, and health physics.

Special covers were made for a Crooke's tube to permit clear demonstration of important phenomena, and the sensitive image orthicons could easily pick up the familiar Maltese cross shadow and the magnetic deflection of an electron beam. The television camera lens was used, following a diffraction grating, to focus directly on the mosaic of the orthicon spectral lines from a mercury vapor source. It was even found possible to pick up the alpha-ray tracks produced in a cloud chamber.

The close-ups obtained with the use of the television camera give the viewer much better detail of any piece of

apparatus than could be had by a classroom observer, and it would appear that almost any demonstration in physics, if carefully prepared, could be given with absolute clarity through the television medium.

Although the nuclear physics series ended on March 13, the University of the Air continued for another eleven weeks. The writer, however, has found among viewers considerable interest in the physics program. It is, of course, not known what the future has in store for physics on television, but many science programs are now available to television viewers, and we should not close our eyes to any opportunities for the presentation of physics courses. Finally, the experience the writer gained in the application of visual aids in physics has been unequaled.

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Science on Television

EDUCATORS have rather recently awakened to the tremendous potential of television as a means of disseminating knowledge. Barely in time, we are putting in a plea for a reasonable share of the few remaining possible channels.

Science has not been televised very much, principally because managers of commercial stations feel that the public prefers news, variety shows, comedy, music, plays, and mystery films to science. A few notable exceptions stand out. The work of Dr. Roy K. Marshall in his "Nature of Things" and his short science experiments for the Ford Hour have been excellent. Dr. Thomas P. Merritt of Albright College gave a fine series of programs on nuclear physics. "Mr. Wizard" (Don Herbert) is continuing to explain many ideas of science by his admirable television experiments.

In Detroit, as early as 1949, WXYZ-TV put on a station-sponsored program, "Magic of Science," with the author as master of ceremonies. This was a thirteen-week series of fifteen-minute programs, mainly on physics principles, but with a few on chemistry and astronomy. The titles of a few will indicate the kind of subjects televised: Stability, Tricks with Air Pressure, Mysteries of Bird Migration, Liquid Air Experiments, Glassblowing, How You Can Make a Telescope, Projectiles, and Sound Experiments.

Under the sponsorship of Hallicrafters Television, the author had an opportunity to show a 26-week series of short science experiments with brief explanations. At first thought, it might be doubted that anything worth while could be done in one minute and forty seconds! However, with careful planning and rehearsing, I found it possible to present many worth-while ideas in science. The following topics were among those covered experimentally and explained reasonably well, if briefly: strong magnets, electromagnets, the earth as a magnet, air pressure, barometers, the cartesian diver, convection, action of Christmas tree bubble lights, conductivity, expansion, rocket principle, center of gratity, chain reaction, phases of the moon, cause of seasons, electrolysis, static electricity, lightning, and lightning rods.

In the spring of 1951 many television stations cooperated with various colleges and universities in putting on educational programs. The University of Michigan presented a carefully planned series on human biology in cooperation with WWJ-TV, followed by a series on photography.

Wayne University was invited to work with WJBK-TV in putting on half-hour educational programs five days a week for a thirteen-week period. Wednesday was chosen as science day, and it was the responsibility of the author to have a 30-minute science program ready to go "on the air" each Wednesday at 1 P.M. An appeal was made to science staff members to volunteer to put on such programs. Most of the subjects were from physics, partly because so many physics lecture experiments can readily be televised, and partly because most of the Wayne University physics staff had had previous experience on the "Magic of Science" programs and are very television-minded. Two excellent chemistry programs were given, "Carbon in the Home," and, "Chemical Elements in the Home"; geology was represented by a fine program on "Gems." We found that in thirty minutes we could present from ten to twenty related experiments on a general topic. The first of the series, "Water—From the Ocean to You," was presented jointly by a physicist, a chemist, and a geologist, and was conclusive evidence that television is a good place to cut across conventional science boundary lines. This was followed by "Water, Steam, and Ice in the Home," "Heat Transfer in the Home," and "Control of Moisture in the Home." These four formed a well-integrated sequence, however, since the series was experimental in nature, it was decided to give up continuity in order to try out various topics and sciences on television.

One of the most noteworthy science television programs was the lunar eclipse of April 12, 1949. This was done by Robert H. Esling and Everett R. Pehlps over WWJ-TV in Detroit. To obtain a large image of the moon the eyepiece of a 6-inch reflecting telescope was "racked out," as is done on getting an image of the sun on a cardboard for showing sun spots, but instead of allowing the image to be formed on a white card, the rays were allowed to fall on the mosaic of the television camera, and, thus, the image of the moon was sent out over the air and received on thousands of sets. The program ran a full hour; we mixed shots of the eclipse obtained as described above with smaller views obtained through a telephoto lens and with demonstrations in the studio of causes of eclipses using a planetarium and a sotellunium.

Weather programs should be classified as science, particularly if the "weatherman" makes an effort to explain the principle causes of winds and weather. A pioneer in this field was Mr. Armand Spitz, who gave weather over one of the Philadelphia stations for a period of three years. During the spring of 1950, the author gave over WXYZ-TV, "Tomorrow's Weather," for the country as a whole, and for Detroit and vicinity in particular.

At present, the author is the "Taystee Weatherman" over WJBK-TV in a program sponsored by the Purity Bakeries, Inc. A map of the United States covered with a transparent plastic is used, and with a felt pen the various highs, lows, fronts, and precipitation expected over the country as a whole are shown. The short program

is concluded with a specific forecast for Detroit and vicinity. The same company sponsors weather programs in five other cities.

At the meeting of the American Association of Physics Teachers held at Michigan State College June 26–28, 1951, the author was asked to present a paper on "Science on Television—Demonstrations." Because of the writer's previous commitment on the weather program, his colleague Mr. Edward R. Bascom agreed to put on the demonstration which had been worked out together as a joint effort.

Through the splendid efforts of Dr. Armand L. Hunter and Mr. Davis of the Michigan State College television staff, a closed-circuit telecast was arranged. The audience was able to see Mr. Bascom perform the experiments at the lecture table and at the same time watch the results on one of the several television sets located at strategic places in the auditorium. This enabled us to demonstrate that many experiments can be seen better as close-ups on a television screen in one's home, than by a person sitting half-way back in the auditorium where the experiment is being done.

Experiments were chosen from several fields of physics to indicate the wide range of experiments and topics that can be successfully televised. Mr. Bascom began by showing two simple but rather surprising experiments on center of gravity, then some on air pressure including the spectacular experiment on getting a hard-boiled egg into a milk bottle and out again without breaking the egg (or bottle!). Convection currents were made visible by using fuming titanium oxide. Mr. Bascom then demonstrated the flow of hot water through a special hot water heating system he built for the television program.

A series of experiments illustrating Bernouilli's principle came next—a simple atomizer, electric light bulb supported on a jet of air and a screwdriver whirling in a jet of air. Incidentally, the lack of a compressed air line in the auditorium was overcome by using an air compressor from the college paint shop.

The production of sound from a bowed glass tumbler and nodal patterns on a Chladni plate were demonstrated. Resonance pendulums and resonance forks showed very well.

Finally, some simple experiments with the electric generator and electric motor concluded the televised part of the program which was of exactly thirty minutes duration.

Let us consider briefly the future possibilities of educating the public in science topics by means of television. In the first place, I am convinced that we should not try to televise present classroom courses. Interesting as is the subject of physics, if we developed the usual mathematical aspects, we would lost most of our audience in a hurry. I do not mean that we should be content with an unrelated series of programs. We should present our science in proper sequence, but it will be necessary to make it relatively nonmathematical; experiments should be used even more extensively than in our conventional lectures. An attempt should also be made to cross freely the boundaries between sciences.

I suggest that science teachers, especially physics teachers, who reside in or near areas where there are television stations become acquainted with the station managers and do what you can to promote science education over television.

Of course, the big hope for giving the public an opportunity to understand science through the medium of television lies in the colleges and universities obtaining several of the remaining wavelengths. Let us all give whatever backing we can in this direction, and when the channels are obtained, it will be the responsibility of science teachers to do the necessary work and make the necessary adaptations in their methods of teaching to insure the success of science education via television.

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The Starting Point in Physics Teaching

IN a recent article[1] on "Teaching by publication," L. W. McKeehan, in a section referring to modern physics textbooks, makes a remark to the effect that the starting point for teaching the physics of particles is "in electron guns and oscilloscopes," and that students should be "worked up to macroscopic mechanics as a special case." Although this statement is tangential to Professor McKeehan's well-considered plea for more lucid publications in physics, it is in my opinion based on so apparent a misconception of the learning process and is so inappropriate a recommendation to authors of textbooks that I would like to bring the opposite point of view to the attention of your readers.

Virtually all teachers distinguish between the student who learns by rote or memorization and the one who attains a true understanding of the subject. Most of us favor the latter variety. In my experience the distinction rests largely on whether or not the student learns by reasoning from things known and familiar to phenomona and theories more remote from ordinary experience. One of the greatest defects of present textbooks in physics (as well as in other sciences) from this point of view is that they present the subject in so pat a form and from a starting point so far removed from the average student's immediate experience that the student is forced to memorize without thought or reason. In these books, it is bad enough that such concepts as velocity, acceleration, force, mass, vector, and rate of change are taken as a starting point and that so little time is spent in clarifying their meaning. How much worse would it be to start from electron guns and oscilloscopes when the student has no idea where the concept of an electron came from, much less how one interacts with the controlling electrical and magnetic fields in an oscilloscope!

The situation is well illustrated by the fact that most physics students have learned that the earth is round and about 8000 miles in diameter, yet a majority cannot give a simple explanation of why they believe this to be the case except that some textbook writer said so.

For some years, in an introductory course at the University of Chicago,[2] we have stressed the importance of the fundamental concepts of physics, not by presenting them in easily memorizable form, not by devising logically perfect definitions which appeal to a mature scientist and

have but little meaning to a student with a year or less of background in scientific observation and theory, but by discussion of the publications in which these concepts were developed, by discussion of the need of the concept and usefulness of physical laws involving it in treating the phenomena to which it was first applied and of the developing structure of physical theory through the years.

If the study of what Newton and his contemporaries had to say about force, momentum, energy, and inertia is "to bow to the effigy of Newton," as Professor McKeehan would have it, then such bowing (and to lesser figures) must be admitted to be the main activity of the scientific scholar who studies more modern and less carefully written literature in order to understand the concepts of nuclear shells or of interstellar acceleration of cosmic rays.

It is a mistake to assume that this approach applies to elementary or survey-type courses alone. It is my experience, again, that intermediate and advanced courses can be taught most effectively and with little or no extra difficulty by assigning the students individual broad topics for library research, to be reported in seminar-type class meetings and written up as term papers. A textbook can be specified for purposes of linking together these topics, for reference, and for final review. This technique has stimulated students to do more work; it has trained them in independent work, in verbal and written presentation of ideas, and in criticism of the presentation by their peers. If the starting point is not clear, the student is forced to search for earlier or less advanced articles or books. In this manner the weaknesses of textbooks are automatically neutralized. The weaknesses of review articles, which Professor MeKeehan points out, soon become apparent when students find they cannot justify a conclusion from the review alone and must, in many cases, seek out the original papers.

THORNTON PAGE

University of Chicago
Chicago, Illinois

[1] L. W. McKeehan, Am. J. Phys. 19, 9 (1951). The passage quoted is on p. 11, top right.
[2] *Introductory General Course in the Physical Sciences* (3 volumes), edited by Thornton Page, chairman, and members of the College Physical Science Staff (The University of Chicago Press, Chicago, 1949).

Radioactive Disintegration

THE conventional development of parent-daughter relationships in radioactive disintegration follows the treatment given in Rutherford, Chadwick, and Ellis[1] in which a trial solution of the differential equations appropriate to the process is proposed, and the arbitrary constants are evaluated by application of boundary conditions. At the senior-graduate level at which the first course in nuclear physics is usually taught, the following derivation is believed to be considerably more instructive.

Writing unprimed symbols to represent parent nuclei, primed symbols to represent daughter nuclei, and calling N_0 the number of parent nuclei of disintegration constant λ present at time $\tau=0$, the number of daughter nuclei formed in a time interval $d\tau$ at time τ is $\lambda N_0 e^{-\lambda\tau}d\tau$. The number of daughter nuclei remaining at a later time t will be

$$dN' = (\lambda N_0 e^{-\lambda\tau}d\tau)e^{-\lambda'(t-\tau)},$$

and the total number of daughter nuclei on hand at time t may be found by summing the contributions from all $d\tau$ intervals from time 0 to time t:

$$N' = \int_0^t \lambda N_0 e^{-\lambda\tau}e^{-\lambda'(t-\tau)}d\tau = \left(\frac{\lambda}{\lambda'-\lambda}\right)N_0(e^{-\lambda t}-e^{-\lambda' t}).$$

Reinterpretation of the symbols N, λ, t as intensity, absorption coefficient, and thickness results in the determination of the intensity of fluorescent radiation from an absorber where the unprimed symbols represent incident radiation and the primed symbols represent fluorescent radiation. The resulting equation has been reported by Insch[2] in connection with the calibration of proportional counters.

ROBERT KATZ

Kansas State College
Manhattan, Kansas

[1] Rutherford, Chadwick, and Ellis, *Radiations from Radioactive Substances* (Cambridge University Press, London, 1930), p. 11.
[2] G. M. Insch, Phil. Mag. 41, 857 (1950).

Designation of a Thyratron Tube

IN a recent article by Rudy[1] on the excitation potential of mercury the FG-67 thyratron tube is recommended. The nomenclature of this tube has been changed. The General Electric Company now calls it a GL-5728/FG-67. RCA manufactures a similar tube which is known as 1904. This latter is available on the war surplus market at a considerably lower price than either of the above at the list price of around $20.00 each.

CARL C. SARTAIN

University of Alabama
University, Alabama

[1] W. Rudy, Am. J. Phys. 16, 188 (1948).

Fellow Members Please Read the Fine Print

IN October, 1950, a letter was sent to all members of AAPT. Enclosed were (1) an invoice for dues clearly payable to the American Institute of Physics as collecting agency and (2) an envelope addressed to the undersigned as Chairman of the Committee on Nominations. The letter itself did not explicitly state that checks were to be sent to the American Institute of Physics. It did specifically request that the membership submit names of persons considered "as worthy officer material" in the enclosed envelope. Do physicists read the fine print? Obviously not! The results to date (December 19) are as follows: 405 checks have been received by the Nominating Committee. Ten of these enclosed nomination blanks. A total of 110 nomination blanks were returned, 90 percent of which were received approximately on time.

Two hundred and ninety-three different persons were suggested as worthy officer material. Two hundred and eight of these were mentioned once. Fifty-two were mentioned twice, twelve were mentioned three times, and approximately twenty members were listed four times or more. From this list the committee has selected candidates for the annual election slate. It is encouraging that we have so much (10 percent of all) "worthy officer material."

We solicit more active participation by members in making suggestions to the Nominating Committee. The Secretary has assured me that rather more explicit instructions will be given in future.

FRANCIS G. SLACK, *Chairman*
Committee on Nominations

Vanderbilt University
Nashville 4, Tennessee

ANNOUNCEMENTS AND NEWS

Daniel Leslie Rich, 1879–1951

The Association of Physics Teacher and the whole community of physicists has lost a loyal and valued colleague in the death of DANIEL LESLIE RICH on February 15, 1951, at Ann Arbor, Michigan.

DR. RICH was a native of Pennsylvania, where he obtained his preparatory and college training. He graduated from Waynesburg College in 1902, and began his career as a teacher in the high school at Edgewood Park, Pennsylvania. Two years later he became the principal of the high school at Braddock, in the same state. In 1908 he entered the University of Michigan and in 1909 was appointed Instructor in Physics. He served the University continuously until his retirement as Emeritus Professor in 1949.

DANIEL LESLIE RICH

In his forty years of active teaching he endeared himself to thousands of students, many of whom will recall his lectures in general physics as one of the high spots in the college program. He had an unusual talent for devising pertinent and intriguing demonstration experiments. His keen wit and his unfailing enthusiasm, and most of all his sympathetic interest in students and their problems, marked him as an extraordinary teacher.

He was interested not only in teaching, but also in teachers. He frequently offered during summer sessions courses designed especially to assist and stimulate high school teachers of physics. In the course of years he visited a large fraction of all the high schools in Michigan, freely offering appreciation, advice, and constructive criticism. As an enthusiastic supporter of the Michigan Schoolmaster's Club, and of the Michigan Teachers of College Physics, and as a frequent contributor to their programs, he exerted an influence on education which would be hard to measure.

Retirement for DR. RICH by no means signified the end of his professional interests. From the beginning of his last furlough in 1948, he carried on for two and a half years extension lectures in Detroit under the University's adult education program; also, he appeared many times in television broadcasts, giving demonstration lectures on everyday physics. Thus he succeeded in rounding out to its very end a long and fruitful career.

E. F. BARKER

Book Reviews

Molecular Spectra and Molecular Structure. I. Spectra of Diatomic Molecules. GERHARD HERZBERG. Second Edition, Pp. 658+xv, Figs. 207, Tables 39. D. Van Nostrand Company, Inc., New York, 1950. Price $9.75.

Herzberg's book, which for the last decade has been the almost unanimous choice as a textbook in graduate courses and also widely used as a reference book, has now appeared in its second edition by a different publisher. The first edition was published as the last among monographs on diatomic spectra at the time when the rapid advancement of spectroscopy slackened its pace. The wide acceptance, however, and the long lasting popularity were due not only to its timely appearance, but also to the great skill and clarity in presentation of the subject and to a good balance in discussion of the experimental and theoretical problems.

Assuming that most of the readers will have a knowledge of quantum mechanics, Herzberg has added in the new edition many sections treating more exhaustively the theoretical aspects of molecular spectra. Furthermore, for the benefit of the more advanced spectroscopists, much material has been added at scattered places concerning more special problems either left out from the first edition or containing the results of more recent work. Complete tables of all known spectroscopic constants for diatomic molecules in the normal and excited states brought up-to-date to the end of 1949 will no doubt be of great help to anyone looking for such data. Interesting new figures have been added, like the one on eigenfunctions of a symmetric top (Fig. 57), a Fortrat diagram illustrating the formation of "extra" heads (Fig. 79), a level diagram for triplet transitions (Fig. 125), a new level diagram for the N_2 molecule (Fig. 196) and a number of others. In general, an excellent editing job was done—in only one formula a printing mistake was noticed by the reviewer. Figure 101 has been reprinted without correcting an error in the lowest j value.

According to the experience of the reviewer, the most difficult section for students to understand and digest was the section on the symmetry properties of rotational levels. In the new edition this section has been considerably improved by many changes and complete rewriting of whole

paragraphs. In spite of Herzberg's effort to bring up to date also the chapter on continuous and diffuse spectra (Chapter VII), it still remains the least complete part of the book. A few incorrect statements made in this chapter are probably due to the fact that Herzberg was not using original sources. References given at the end of the book are not always made to the most important papers on a specific question. A careful reader will be puzzled, for instance, by the content of the Sec. (f), Part 2 of Chapter VII, which was reprinted with only small changes from the first edition: are the spectra treated there in any way different from those discussed in Part 1, Chapter VII? It has been shown that the explanation of the structure given in Sec. (f) is oversimplified, the diffuseness being, at least in the best known case, of an entirely different origin. It would seem better, therefore, to have left out Sec. (f) completely and introduced a discussion of the influence of the noncentral collisions on the structure (topic missing in the book) together with other details about fluctuation bands in absorption and emission in Part 1, Chapter VII.

Herzberg's book, which already in its first edition seemed a little too bulky and too specialized as a text for an average graduate course, now becomes something of a problem for a teacher of such a course. It must be emphasized that for a person not acquainted with the theory of molecular spectra this book with its great concentration of specific information does not represent easy reading. On the other hand, leaving out 50 percent of the text or even more always affects adversely the continuity of the course, in spite of the fact that Herzberg divided the text very skillfully into two distinct parts by using small type wherever a section was intended for the more advanced reader. Furthermore, Herzberg's effort to cover most of the specialized problems has led to an overcrowding of many sections with names and more recent, and in some cases meticulous, references. Thus a nonspecialist might lose the perspective so necessary in a good graduate textbook. The reviewer feels, therefore, that as a result of the change in the character of Herzberg's book, there is an urgent need for a comprehensive textbook on molecular spectra, which should cover the fundamentals of diatomic spectra and polyatomic as well, without leading into a labyrinth of intricacies, and which could be covered in a one-semester course. If we do not give such courses regularly at an early stage of graduate studies, how can we expect to raise a new generation of spectroscopists? Even when, after successfully solving the main problems in polyatomic molecules, spectroscopy shall have become a field worked out to a high degree, good spectroscopists will still be needed as much as specialists are needed in other classical fields of physics.

S. MROZOWSKI
University of Buffalo

Laplace Transformation. WILLIAM T. THOMSON. Pp. 230 +ix. Prentice-Hall Inc., New York, 1950. Price $3.75.

In Professor Thomson's work we have yet another textbook on Laplace transforms which is designed for use in American universities. Other texts on the same subject include those by Gardner and Barnes, Churchill, and Goldman. It is to be noted that all of these works have appeared since 1942, which reflects the fact that the existence of courses on Laplace transformation methods is a recent innovation in American curricula. The books by Gardner and Barnes and by Goldman were written primarily with the electrical engineering student in mind. Churchill wrote primarily for the applied mathematician, but Thomson has written for the mechanical engineer. Could it be that soon an American textbook will appear which is aimed at physicists?

This book by Professor Thomson compares favorably with others that have appeared in this country. Particularly to be commended is the more extensive use of theorems, which can shorten calculations tremendously, than is common in other texts. Illustrative examples have been chosen carefully and add much to the understanding of both the general technique and the theorems. The two hundred nineteen problems scattered throughout the text are excellent.

In his presentation of the subject, Professor Thomson quickly lays the mathematical foundation for the treatment of ordinary differential equations. This is followed by two chapters on applications to dynamical and structural systems. A chapter on complex variable theory is then inserted in order to handle the material in the following chapter on partial differential equations. The book closes with chapters devoted to difference equations, closed loop systems, and electromechanical analogies.

The book is very appealing to mechanical engineers and should enjoy a wide adoption. Users must realize, however, that a treatment of Laplace transforms, with illustrative material, which is limited to 226 pages must be brief in its explanations and even omit important material. It is in order, therefore, to make some comments for the benefit of teachers who may use the text.

In the first place, the Laplace transform is popular mainly because the concept of a transformation from a real to a complex variable is more appealing to the scientific mind than the operator concept of Heaviside. To students who have never had contact with Heaviside's methods, the direct and inverse Laplace transformation seems highly artificial, because they make no contact with past experience. Such a contact is easily made and should be presented always.

The author defines the direct Laplace transform as follows: "If $f(t)$ is a known function of t for values $t>0$, its Laplace transform $\bar{f}(s)$ is defined by the equation

$$\bar{f}(s)=\int_0^\infty e^{-st}f(t)dt,$$

and abbreviated as

$$\bar{f}(s)=\mathfrak{L}f(t).$$"

Equivalent statements are found in the other texts cited. The reviewer has found that students experience difficulty in grasping the real translation theorems if they are explained on the basis of the above definition. This difficulty disappears if at the outset students are told that the transformation is applied to a function of the form $f(t)\cdot u(t-a)$, $a\geq 0$. This point of view can be justified theoretically.

In so brief a book, the author can merely mention the abscissa of convergence, a quantity which plays a fundamental role in the theory and understanding of the method. A user of the book should supply this additional information. He should also complete the treatment of Jordan's Lemma in Appendix B.

It is further recommended that students be taught multiple transformation techniques in dealing with systems describable by partial differential equations, for this method can lead to great simplifications in certain situations.

The reviewer prefers the Routh-Hurwitz technique, rather than the Nyquist diagram, for checking on the stability of systems, although the latter has been stylish for some years. If stability is to be determined by a Nyquist diagram, it is recommended that it be explained by showing that the number of times a plot of $F(z)$ encircles the origin with z following a specified closed contour, C, is equal to the multiplicity of zeros of $F(z)$ inside C diminished by the multiplicity of poles. The usual treatment, which the author follows, involving the contour integration of the logarithmic derivative of $F(z)$ is confusing to students.

In the last chapter, on electromechanical analogies, it is unfortunate that the author has employed the order of explanations there used. He explains that there are two analogies in common use which he designates as the force-voltage and the force-current analogies. He then presents rules for constructing the force-voltage analogy. Two methods are given for constructing the force-current analogy; the first calls for the construction of the topological dual of the network found by the force-voltage method, the second, and preferred one, proceeds directly from a connection diagram of the system to the equivalent electric network. In general, it is desirable for each component in the equivalent electric circuit to have a one-to-one correspondence with the elements of the mechanical system. This corespondence cannot be maintained in a force-voltage analogy if the connection diagram of the mechanical system is topologically nonplanar but it can be realized always on the force-current analogy since the topology of the connection diagram and of its equivalent electric network are identical. Hence, it would have been better if the author had started with the force-current analogy and had obtained the force-voltage analogy, when it exists, by dualization.

The author's treatment of scaling procedures by means of the Buckingham π-theorem is good.

It is hoped that the above suggestions may be useful to teachers who plan to base a course on Professor Thomson's book.

HORACE M. TRENT
Naval Research Laboratory

Physics, Its Laws, Ideas, and Methods. ALEXANDER KOLIN. Pp. xv+890. Figs. 460. 16×23.5 cm. McGraw-Hill Book Company, Inc., New York, 1950. Price $6.50.

The first feature of this book which impresses me is its weight, 1.57 kg. The weights of some other heavy textbooks of general physics are: Lemon and Ference, 2.44 kg; Margenau, Watson, and Montgomery, 1.46 kg; Stewart and Gingrich, 1.1 kg; Hausman and Slack, 0.95 kg. Many older textbooks weight less than 1 kg. Let us hope that the physical as well as the mental abilities of our students is keeping pace with the growth of the material which we wish to include in general physics. We are all aware of this problem of selecting material. But the reader of this review presumably wishes to know whether the selection and method of treatment in this book is suitable for his own classes.

The author states that his course has a double purpose:

1. To satisfy the requirements of the Division of Biological Sciences in regard to the training in physics of students entering that division.
2. To satisfy the general education objectives of the college in regard to the training of students in the methods of the physical sciences.

It is evident that the author is striving to attain these objectives. The book contains many excellent examples of the application of physics to biology. Many questions at the ends of the chapters deal with biological problems.

The second aim stresses methods, and, I may add, the development of our ideas. The order of topics is not conventional. The student is led from the concrete to the abstract—from mechanics, through subjects using mechanical models, through wave theories and electromagnetic fields, to the greater abstractions of quantum mechanics and relativity. This emphasis on ideas and theories may have caused the lack of emphasis on physics as a part of our everyday life. Some problems are quite unrealistic in their choice of data. For example, in the conduction of heat through a windowpane, the student is led to assume that the surface temperatures of the glass are approximately the indoor and outdoor air temperatures—an assumption which leads to a heat loss perhaps thirty times too great.

The book appears to give a good sound treatment of physics. There are, as one can always expect, a few mistakes and misstatements of fact which may be corrected in a later printing. There are many excellent diagrams and photographs. Some of the following features of the book are distasteful to me but may be approved by others.

There are too many words. An example of this is the paragraph on kinematics which includes this sentence, "The study of changes in spatial disposition of geometric elements in the course of time is the subject matter of kinematics." If the word kinematics needs to be introduced at all, would it not be as well to say only that it is the study of motion? It seems probable that the book could be shortened by twenty percent without any loss in clarity, but the task of doing this would not be easy. It could also be shortened by omitting many topics and explanations which will not be understood by the student.

In electricity the cgs electrostatic, electromagnetic, practical, and mixtures of these units are used. I deplore this. The student could be spared some useless work if the newer mks units were used. The newer approach to magnetism, with less emphasis on poles, would also be desirable.

Snell's law is written in the form $\sin i/\sin r = n$ rather than the preferred form $n_1 \sin\theta_1 = n_2 \sin\theta_2$.

The terms inertial mass and gravitational mass are used, although mass is carefully defined in the former sense. The phrase "inertia of masses" has also slipped into the text.

D'Alembert's principle should not be introduced into a first course in physics. It merely serves to confuse a student until he has first learned to isolate a body and analyze the forces acting *on* it.

Many of the objections which I have raised are on controversial issues, and there is no text without faults. On the whole, this one should rank high, particularly for the biology student and the student who will not make practical use of physics.

Many physicists and educators have been concerned with the problem of giving the educated layman, the professional man, and the business man some understanding of physics and its relation to our present civilization and culture. Most one-year physics courses in liberal arts colleges are presumably for this purpose. This book shows a new approach to the problem, and we will be interested to see how successful it is. The student will not find it all easy reading. For one thing he must learn an extensive vocabulary. There are some nine hundred entries in the index and most of these involve either new words for the nonscience student or old words with specialized meanings. This is more of a burden than we often realize, and it is in addition to gaining new concepts, learning new principles, and understanding new theories. This book does not, however, demand that the student use mathematics beyond simple algebra (except for the occasional use of a sine and cosine). There is little emphasis on the quantitative aspects of physics as they might be applied to practical situations in the student's experience. The quantitative problems, as contrasted with the qualitative questions, are largely exercises in algebra and substitutions in formulas.

W. H. MICHENER
Allegheny College

Chemistry Visualized and Applied. ARMAND JOSEPH COURCHAINE, Instructor Biological Chemistry, Hahnemann Medical College, and Science Instructor, Hahnemann Hospital School of Nursing, Philadelphia, Pennsylvania. Edited by M. CORDELIA COWAN and illustrated by RICHARD ALBANY. Pp. 687+xiv, Figs. 122. 14×22 cm. G. P. Putnam's Sons, New York, 1950. Price $5.50.

"This book is designed primarily for students contemplating or preparing for a career related to health and disease···." It is composed of three parts:—Fundamentals and Applications of Inorganic Chemistry—12 chapters, 262 pages; Fundamentals and Applications of Organic Chemistry—8 chapters, 132 pages; Introduction to Biological Chemistry—10 chapters, 227 pages.

Obviously, no pains have been spared in the mechanical preparation of this textbook. It is well printed on good stock with a good binding. The illustrations are profuse and include all kinds, photographs, excellent and often clever drawings, many tables; structural formulas are liberally used.

The plan of the text has been well thought out, and many helps for student and teacher have been incorporated. Some of these include suggestions to both students and teacher, introductory questions at the beginning and test questions at the end of each chapter. The tone of these questions at times sounds like a catechism rather than a demand for the understanding which the author lists as one of his objectives. The text is perhaps most remarkable as a condensation rather than a judicious selection of the material which might be presented.

The most regrettable features of the book are the misstatements of fact and false implications. These are too numerous to list completely, but the following are examples: "Always bear in mind, as you progress in your study of chemistry, that there are only four general types of compounds—oxides, acids, bases and salts" (p. xii). "The valence of nitrogen in ammonia is −3, but it is +5 in ammonium chloride" (p. 33). "The heat produced when a substance is dissolved in water is known as positive heat of solution, and is not to be confused with exothermic reactions, in which heat is liberated during a chemical change" (p. 175). In chapter 10—Salts—molecular equations are consistently used to represent ionic reactions. Under the heading Normal or *Neutral Salts* sodium carbonate is listed. On p. 207 sucrose is listed as hydrolyzing to form *two* molecules of glucose; also, "When copper sulfate hydrolyzes in aqueous solution, it produces a strongly acidic reaction, since a strong acid, sulfuric acid, H_2SO_4, is formed." And again, "According to Raoult's law, the depression of the vapor pressure is proportional to the number of molecules of a solute dissolved in a given volume of solvent. Thus, by doubling the number of molecules of solute, *the vapor pressure of the solvent is doubled*" (p. 231). On p. 233 and elsewhere covalent compounds are listed as being nonpolar. "Covalent compounds such as ammonia, NH_3, and hydrochloric acid, HCl, *contribute* a proton to solvent (water) molecules by coordinate covalence···" (p. 249). On p. 331 sodium ions are represented as combining with carbonic acid molecules to form hydrogen gas.

It is unfortunate that the same care that went into the mechanical makeup of this book has not been matched in writing the body of the text.

FREDERIC B. DUTTON
Michigan State College

Physics. S. G. STARLING and A. J. WOODALL. Pp. 1301 +xvi, 22×15×7 cm, 1760 g. Longmans, Green and Company, London and New York, 1950. Price $8.50.

The English custom of publishing textbooks of physics is usually to have special books for each subdivision of the subject, e.g., properties of matter, heat, and light, especially for the higher grades. Inclusive books of elementary standard are issued, but even these are sometimes available in separate parts for the convenience of students. The American custom was to issue a multiplicity of general

textbooks of an elementary nature, which were largely descriptive and often included formulas without proofs. Recently, however, there has been a welcome departure from this custom, and now we have many American books on the different subdivisions of physics. For example, to mention a few I have on heat, there are the books of Zemansky, of Worthing and Halliday, and of Weld.

The present book by Starling and Woodall is a huge book and is sufficiently full in its treatment to be claimed as advanced. The writers say it is intended for the general honors and pass degrees in English universities and presumes as a prerequisite a knowledge of physics up to the "Intermediate" standard, which means a standard about one-third of the way from matriculation to degree.

How would this book suit the requirements on this side of the Atlantic? In Toronto we have a four-year honor course of mathematics and physics. This course starts with a class of about 90 in the first year and the physics taken in this year is mechanics, properties of matter, and heat. In the second year light, sound, and electricity and magnetism are taken. The third and fourth years carry these subjects to higher levels and introduce the "modern" physics. Such a course gives the students comradeship and a spirit of emulation. It suits such students to use special books for the different subdivisions of physics, one reason being that these books are portable. Would the book under review meet their wants? As a reference book for correlative reading it should prove useful. Taking the sections in seriation, there are 121 pages on general physics, 307 on heat, 371 on light, 127 on sound, 308 on electricity, and 66 of tables and problems.

The treatment in general follows the historical sequence, but there are practically no biographical details of the chief workers, and, thank goodness, no photographic reproductions of the same men, usually at an advanced age, as in some books. Naturally, the treatment does not rise to the level given in the advanced English textbooks on the different subjects, although in places it may run pretty close.

Taking the section on general physics, one misses in the chapter on gravitation a treatment of the acceleration of gravity and its variation over the surface of the earth, and also a critical study of the work of Boys and others, and of Heyl in particular, on the gravitation constant. Neither is there any mention of the use of gravity measurement in geophysics. The chapter on elasticity is very brief, elementary and disappointing, no mention being made of the St. Venant problem in torsion and of the relation between the elasticity of a material and its structure (as revealed in x-ray analysis), nor of the variation of these constants with temperature and the past history of the specimen. Possibly the authors think that this is the job of an engineering textbook.

Viscosity has a better treatment, having a section on thixotropy, but still the treatment is not sufficiently critical; to get good results in viscosity one has to remember the assumptions and limitations of the method employed. The chapter on surface tension is uninspiring; nothing is said about the beauty of this particular branch of physics; the treatment is simply a bald study of the methods of measuring the coefficient of surface tension, relieved, however, at the end by a brief mention of the work of Langmuir and Adams.

In fact, right through this treatment of general physics the aim has been to describe the elementary methods of measuring the values of the physical constants without saying why we are interested in these subjects, what they stand for in general life and industry, and how their study helps us to understand—or at least to seek to understand—the secrets of Nature. In particular, in the chapter on surface tension I was hoping to see a discussion of the controversy on whether or not there is an actual surface film in tension or whether, as is sometimes stated, the surface tension is a mere mathematical fiction, and that the only real force is an inwardly directed force due to molecular attractions from the interior. Perhaps having spent one's life on properties of matter and heat, one expects too much on the side-lines.

The treatment of heat is orthodox and elementary. The kinetic theory is of the simplest. The pressure-temperature relation for vapors is given in the form $\log p = A - (B/T) + C \log T$ but no attempt is made to derive it, or even to derive its simple form $\log p = A - B/T$. Here an historical treatment would be of interest, starting with Dalton and coming through Rankine, Regnault, Dupré to Clausius and Clapeyron. The thermodynamic functions are lightly sketched but hardly sufficiently for a student to be able to use any of their consequences. The work on solutions and osmotic pressure is fair for a physics textbook. A redeeming feature is the chapter on statistical mechanics and quantum theory. This will be welcomed by the young student. In sound the second chapter on wave motion in gases and on the surfaces of liquids is recommended.

I am not so well qualified to judge of the merits of the light and electricity sections as I am of the preceding sections; but since one of the authors has written one of the best books of electricity and magnetism, I should judge this section would be equally good. The treatment covers electromagnetic radiation, x-rays, the elementary particles, radioactivity (natural and induced), and the relations between particles and radiation and the newer electronic devices.

The section on light follows the usual pattern. Exceptionally good are the full page plates illustrating interference and diffraction effects.

On the whole, one must say this an excellent handbook giving a fairly good treatment of all branches of the subject and omitting very few items in our stock knowledge of physics. The book is, however, too large and too heavy, and its pages will not stay open. The authors and publishers are recommended to issue it in parts. At present it is a good book for the library.

JOHN SATTERLY
University of Toronto

New Members of the Association

The following persons have been made members or junior members (*J*) of the American Association of Physics Teachers since the publication of the preceding list (Am. J. Phys. 19, 327 (1951)).

Abell, Dewey Francis, International House, Berkeley 4, Calif.

Adams, Raymond Kenneth (*J*), 702 Pine Street, Boulder, Col.

Andrew, Harold Blair, 341 Demain Avenue, Morgantown, W. Va.

Ashby, William Dale (*J*), 313 So. Swain Avenue, Bloomington, Ind.

Beck, Clifford Keith, Physics Dept., North Carolina State College, Raleigh, N. C.

Bedo, Donald Elro, 1127 Arlington Road, Erie, Pa.

Bell, Graydon Dee (*J*), 551 S. Hill Avenue, Pasadena, Calif.

Biser, Roy Hamilton, 1170 Elgie Street, Beaumont, Texas

Bizette, Henri, 2 rue de la Craffe, Nancy, France

Blanc, Albert James, 422 Grand Concourse, Bronx 51, N. Y.

Boblak, Thaddeus Francis (*J*), 1644 West 44th Street, Chicago, Ill.

Bond, Jean (*J*), 66 Admiral Road, Buffalo, N. Y.

Bondelid, Myron Anton (*J*), 117 Park Avenue, Grand Forks, N. D.

Bort, Robert Lee, 814 W. Johnson Street, Madison, Wis.

Boudreau, Joseph Alfred, Jr. (*J*), Church Street, Fiskdale, Mass.

Box, Harold Charles, Transit Road, Williamsville 21, N. Y.

Boyer, William E., 815 N. Elm Street, Greenville, Ill.

Bridges, Albert Payton, Box 1178, Vanderbilt University, Nashville, Tenn.

Bulman, John Bradley, 1615 Stanley Street, New Britain, Conn.

Burger, Robert Mercer (*J*), 41 Vassar Avenue, Providence 6, R. I.

Burnam, Tod Donald, 1209 I Avenue, La Grande, Ore.

Callen, Earl (*J*), Department of Physics, University of Pennsylvania, Philadelphia, Pa.

Campbell, Richard Graves (*J*), 4916 Forbes, Pittsburgh 13, Pa.

Campbell, Robert Wilfred (*J*), 334 Roley Street, Belle Vernon, Pa.

Chappelle, Austin B., Station A, Box 97, Hattiesburg, Miss.

Choate, Robert Lee, Missouri School of Mines, Apt. T-8, Rolla, Mo.

Christie, Dan Edwin, Main Street, R.F.D. 2, Bowdoinham, Maine

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Coon, Richard Llewellyn (*J*), Collegedale, Tenn.

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RECENT MEETINGS

Oregon Section

The fifty-sixth meeting of the Oregon Section of the Association was held at Linfield College, McMinnville, Oregon, on February 10, 1951. The President of the Section, DR. M. A. STARR, *University of Portland*, presided at the sessions.

The program is given below. Abstracts are included in the case of papers whose subject matter was related directly to the teaching of physics.

Efficiency in teaching measurements. WILLIAM R. VARNER, *Oregon State College.*—A course in measurements is unique because groups usually must do different experiments in a given laboratory period. The difficulties can be met by organization, assignments, and lecture demonstrations. Consistent definitions, well-graded problems, and the correct use of physical equations are necessary for efficiency. All equations should be applicable as simultaneous equations, especially in the conversion of units. Basic experiments should be chosen so that the least number for complete analysis of a certain section may be done. "Don't" was said to be one of the most used, but one of the most useless expressions. The occasional recording of a lecture was recommended.

Calibration of an ionization chamber for absolute x-ray beam power measurements. DUIS D. BOLINGER, *Oregon State College.*

Survey of photoelectric theories. JAMES J. BRADY, *Oregon State College.*—This survey was limited to the photoelectric effect from metal surfaces for frequencies near the threshold. A block diagram was shown as a teaching aid in presenting photoelectric theories. The effects of frequency and intensity of the incident light on the number of ejected electrons and their energies were separated by use of four separate compartments in the diagram. Special attention was given Fowler's theory, which is expressed in the form of an equation relating the number of electrons ejected per second with the frequency of the incident light. The failure to obtain agreement between experiment and theory in the case of sodium was cited as an illustration of the need for a further extension of the theory.

The design and characteristics of thin-windowed Geiger-Müller counters. R. L. PURBRICK, LAWRENCE T. CHERRY, AND JAMES F. CARPENTER, *Willamette University.*

Neutron detection methods and the demonstration of a BF_3 counter. DALE MARVIN HOLM, *Oregon State College.*—The problems of neutron detection are somewhat greater than those encountered in most other radiation detection because the electrically neutral neutrons do not directly cause ionization of atoms by collision. Methods of obtaining neutron-induced reactions, which do cause the production of charged particles, are needed before the presence of neutrons can be known. Detectors are divided into two groups: the instantaneous detectors and the delayed detectors. As the name implies, the first group gives an immediate indication of the neutron presence, while some time must elapse before the same in formation may be obtained from the second group. In the first division are ionization chambers, proportional counters, scintillation counters, and, for very fast neutrons, electron multiplier counters. The second group includes cloud chambers, radioactive foils, and nuclear plates. Most neutron detectors use one of the following reactions: absorption of a neutron and the emission of an alpha-particle, absorption and fission, absorption and beta-activity, or collision with a proton which recoils. Each of these reactions causes ionization which can be detected. The local heating on the surface of some materials due to very fast neutrons is the source of electrons for electron multiplier counters. A moderator is usually used to slow the neutrons before they arrive at counters which detect best the slower neutrons. The cross section for neutron capture often varies as the reciprocal of the velocity. To date no one detector has neutrons of all energies, although modifications of some of the thermal neutron detectors enable the experimenter to detect all energies.

Demonstration of the diffraction and interference of sound. EVERETT K. JENNE, *Oregon State College.*—The following demonstrations were performed with the apparatus described by H. K. Schilling:[1] Michelson and Fabry-Perot interferometers, Young's double slit, Lloyd's single mirror, diffraction effects of circular and rectangular apertures.

The status of the Oregon State College cyclotron project. RICHARD R. DEMPSTER, *Oregon State College.*—A brief description was given of the O.S.C. cyclotron and building, and of the progress to date. The machine will have 39-inch pole tips, and is expected to have 32-inch dees and to operate at 14,000 oersteds. With 10.8-Mc/sec, 7.5-Mev deuterons will result; assuming suitable shimming, the machine will yield 15.0-Mev protons at 21.5 Mc/sec. Completion is planned for the summer of 1952.

Penetration into the rarer medium in total reflection. MAYNARD DEAN PEARSON, *Oregon State College.*

Report on the New York meeting. RAYMOND T. ELLICKSON, *University of Oregon.*

Dr. W. W. Dolan, Dean of the Linfield College faculty spoke at the luncheon, which was held in the Linfield Student Commons.

FRED W. DECKER, *Secretary*

[1] H. K. Schilling, Am. Phys. Teacher 6, 156 (1938).

Wisconsin Section

The annual meeting of the Wisconsin Section was held on Friday and Saturday, May 4 and 5, 1951, at the University of Wisconsin. The afternoon of the first day was taken up with a tour of the research laboratories of physics on the campus. Included in the itinerary were visits to the laboratories housing the electron microscope, the mass spectrometer, research projects on cancer, the ultracentrifuge, nuclear physics, and x-ray research. An unusual

feature was the attendance of the group at the regular weekly colloquium of the physics department at which PROFESSOR CHARLES A. BOYD of the Department of Chemistry spoke on "Rockets and Jet Propulsion." At an evening meeting at the Washburn Observatory, Professor C. M. Huffer and Dr. A. D. Code presented a discussion of their techniques for obtaining light curves of eclipsing binaries. These light curves are used for the primary purpose of determining the sizes, masses, and densities of stars. Indirectly, the work leads to the determination of colors and magnitudes of stars used for the detection of clouds of dust and gas in the Milky Way. The visitors were also taken on a tour of the observatory.

Contributed Papers

A balanced-beam, drift-eliminated recording microphotometer. A. H. BUDLONG, *Marquette University.*

A nuclear demonstration model. M. OLSEN, *State College.*

Student experiments with a Lauritsen quartz fiber electroscope. G. KARIORIS, *Marquette University.*

Use of a sonic anemometer in micrometeorology and the study of the heat and moisture balance at the earth's surface. R. A. BRYSON, *University of Wisconsin.*

Methods of dynamic atomic energy demonstration. J. L. KURANZ, *Chicago Nuclear Corporation.*

Vacuum spectrograph and its role in Lamb-Retherford shift. RAY SUCHY, *State College.*

Use of geophysics in groundwater investigations with applications to problems of import to Wisconsin. G. P. WOOLARD, *University of Wisconsin.*

A student experiment on the statistical fluctuations of radioactive decay using a scaler. J. G. GRUNDLE, *Marquette University.*

Demonstrations of phase difference between ordinary and extraordinary beams. J. G. WINANS, *University of Wisconsin.*

Two demonstration lectures were given. The first, by F. K. HARVEY, *Bell Telephone Laboratories*, dealt with the focusing of sound waves with microwave lenses. The second, by J. R. DILLINGER, C. K. MCLANE, and F. A. RODGER, *University of Wisconsin*, was a discussion and demonstration of the phenomena of liquid helium film flow, the fountain effect, supraconductivity, and adiabatic demagnetization. An explanation and demonstration was also given of the Collins helium liquefier.

After luncheon on the second day of the meeting, a business meeting was held, DR. H. H. BARSCHALL presiding. A report of the annual meeting of the AAPT was given by PROFESSOR R. R. PALMER, *Beloit College*, and the following officers were elected for the year 1951–52: *President*, J. R. DILLINGER, *University of Wisconsin; Vice-President*, ARTHUR BARKOW, *Marquette University; Secretary-Treasurer*, MISS MONICA E. BAINTER, *Central State College.*

MONICA E. BAINTER, *Secretary*

Southern California Section

The annual spring meeting of the Southern California Section of the American Association of Physics Teachers was held at Pomona College, Claremont, California, on Saturday, April 21, 1951. It was attended by approximately forty-five members and friends. The morning program consisted of an invited paper, "Equipment for the control of air pollution," by DR. CARL KANTER, Senior Engineer, Los Angeles County Air Pollution Control District; and the following ten-minute contributed papers.

Education?—or merely training?! IX. Potential difference *vs* potential gradient. GEORGE FORSTER, *Pasadena City College.*—Incidents in the author's many years of teaching experience were presented (1) as evidence of the need to do more in our classes than merely teach physics, and (2) as interesting and educationally worthwhile introductions to some parts of physics with which they can be associated. One of the incidents occurred in connection with an insulation problem the solution of which depended upon an understanding of potential difference and potential gradient. The following demonstration emphasized the importance of the latter concept.

Two circular metal plates with insulating rods were mounted horizontally, one above the other, and separated by an air space of uniform thickness so as to form an air condenser *C*. A high potential difference was applied to *C* and adjusted to a value just below that required to break down the air insulation. One end of a long glass plate about half as thick as the air space was then inserted into the air gap of *C* so as to lie on the lower metal plate, while the voltage was kept constant. There were now two condensers in series: *C'*, with air dielectric; and *C''*, with glass dielectric. The cracking which occurred as soon as the glass plate was inserted indicated that the air in condenser *C'* had broken down as an insulator and was now conducting. In a darkened room the air gap revealed numerous small visible sparks. The effect was enhanced by raising the voltage temporarily.

The theory predicting the results, certain conclusions, and two practical applications were included.

High voltage and induction heating demonstrations. RICHARD AURANDT, *John Marshall High School* (introduced by William E. Neuswanger).

A simple polarized light demonstration. CHARLES A. FOWLER, *Pomona College.*—The conventional lecture demstration of polarized light phenomena generally brings to focus on a screen a beam of light which has traversed the various polarizing components mounted upon an optical bench. Such demonstrations are less effective than a laboratory experiment in illustrating the essential ideas for the student because, although the particular phenomena may be evident, the function of each polarizing component is often quite obscure. If the lecturer manipulates the polarizing components in the wide object space of a commercial lecture note projector, the viewers receive from the projected image much the same intimate impressions as the individual laboratory student gains by personally manipulating the components. The method is used to demonstrate polarization by reflection, double refraction, the functions of the quarter-wave and half-wave plates, interference of polarized light, and photoelastic phenomena.

Functional definitions and the meaning of electricity. F. W. WARBURTON, *University of Redlands.*—A current

balance using a straight portion of a long wire is demonstrated, and it is shown how this determines the functional meaning as well as the size of the ampere.

A demonstration of the relationships between ac and dc voltage. ALBERT V. BAEZ, *University of Redlands.*—The equality of the heating effects of ac and dc voltages can be demonstrated with a paraffin photometer. It is instructive to calibrate an ac voltmeter this way and it can be done with a small error. The ratio of peak to rms voltage is found by measuring the least dc and ac voltages which cause a neon bulb to glow.

Alternating current experiments at fifty cents each. DEAN H. HARBOUR, *Compton College.*—The construction of a test board for alternating current experiments at an approximate cost of $5.00 is described. Ten experiments with the boards are suggested, for they have proved successful over several years' experience.

The closed differential pulley. LAURENCE E. DODD, *University of California at Los Angeles.*—The motion of a frictionless differential pulley with the lower wheel of a diameter equal to the average of the upper wheel diameters demonstrates in a striking and rather amusing way kinetic behavior in a system relating linear and angular quantities. The two ends of the cord are wrapped around the two upper axles, so that it unwinds from the larger at the same time that it winds up on the smaller, while the lower pulley slowly descends. If this system, initially at rest, is freed, it undergoes within itself displacements, velocities, and accelerations, both linear and angular, owing to the pull of gravity on the lower pulley unit. Many striking features are pointed out, and the detailed analysis is indicated.

The effect of the discovery of x-rays upon the scientific world as I remember it. W. P. BOYNTON, *Whittier, California.*

New approach to the teaching of physics to premedical students. VERNON L. BOLLMAN, *Occidental College.*—Modern medicine and medical research involve the use of extensive apparatus and techniques based on the basic discoveries of modern physics. Three years ago the need for some modification in the laboratory work in premedical physics at Occidental College to furnish the student with some understanding and appreciation of the importance of modern physics became apparent. The orthodox experiments of the second-semester laboratory were abandoned; and new experiments relating to such topics as the origin and nature of x-rays, natural radioactivity, artificial radioactivity, detection and measurement of particles with counters, properties of neutrons, and biological effects of various particles and radiations were included in a list of fourteen selected experiments. The results of this new approach have been most gratifying.

After luncheon at the Claremont Inn there was presented an invited paper: "Particle Counters" by DR. ROBERT WALKER of the *California Institute of Technology.* Dr. Walker not only reviewed the present status of subatomic particles but also indicated ways in which they are severally detected. A demonstration of the pulses produced by gamma-rays from Co^{60} and from Na^{24} in a scintillation counter was given.

At the business meeting which followed, an honorary membership in the Southern California Section was awarded to Richard Aurandt, a student at John Marshall High School, in recognition of his accomplishments in physics as evidenced by the demonstration he had presented earlier in the day.

The following officers were elected for the year 1951–52: *President,* HARLEY J. HADEN, *Glendale College; Vice-President for Colleges,* CHARLES A. FOWLER, *Pomona College; Vice-President for Junior Colleges,* FRED WYATT, *Harbor Junior College; Vice-President for High Schools;* DOUGLAS H. LOWNDES, *Eagle Rock High School;* and *Secretary-Treasurer,* DAVID F. BENDER, *Whittier College.*

An executive committee meeting concluded the activities of the day.

DAVID F. BENDER, *Secretary-Treasurer*

Indiana Section

The annual spring meeting of the Indiana Section of the American Association of Physics Teachers was held at Franklin College, Franklin, Indiana, on May 12, 1951. Thirty-six members representing fourteen institutions attended. Reports on the National Meetings of AAPT were given by D. E. ROLLER and A. D. HUMMEL at the luncheon. A luncheon and garden party was also enjoyed by the wives of members. The 1952 meeting will be held at the Purdue Center, Indianapolis, with M. D. ADAMS, Chairman. The following program, accompanied by unusually spirited discussion, made a very full day's meeting for the Section.

Panel Discussion

Modernizing and improving the undergraduate physics curriculum.

Modern instrumentation needed at the undergraduate level. M. E. HUFFORD, *Indiana University.*

The place of original work in the undergraduate curriculum. D. E. ROLLER, *Wabash College.*

The place of recent modern physics in the curriculum. A. D. SPRAGUE, *De Pauw University.*

Special courses needed for nonscience majors. J. F. MACKELL, *Indiana State Teachers College.*

The minimum curriculum for small departments. R. E. MARTIN, *Hanover College.*

Contributed Papers

Laboratory experiments for an advanced undergraduate course in radiation physics. C. M. ZIEMAN, *Wabash College.*

Another attack on gamma (C_p/C_v). O. H. SMITH, *De Pauw University.*

A Boyle's law—Dalton's law problem solved with gauge pressures. Centrifugal force with sound effects (demonstration). M. D. ADAMS, *Purdue Center, Indianapolis.*

Voltage measurements. R. C. PATTON, *Valpariso Technical Institute.*

Deflection in cathode-ray tubes. J. W. ALINSKI, *Valpariso Technical Institute.*

Resistance network calculations. Physics and TV. An experiment with chalk. Cooling the professor. A. D. HUMMEL, *Ball State Teachers College.*

AC theory and practice. J. F. MACKELL, *Indiana State Teachers College.*

Apparatus for demonstrating osmosis. H. D. SMITH, *Purdue University.*

Models for Rutherford and Thomson scattering. I. WALERSTEIN, *Purdue University.*

A discussion on what the graduate schools expect of the undergraduate physics training was presented by I. WALERSTEIN of *Purdue University* and E. J. KONOPINSKI of *Indiana University* at dinner.

J. C. HENDRICKS, *Secretary*

Kentucky Section

The annual spring meeting of the Kentucky Section of the American Association of Physics Teachers was held on April 14, 1951, in the Physics Building of the University of Louisville. The meeting, held in conjunction with the annual convention of the Kentucky Education Association, was attended by 40 members and guests. DR. WALDEMAR NOLL, *Berea College*, presided. The program consisted of five contributed papers as follows.

Lenses for spectrographs. RALPH A. LORING, *University of Louisville.*

Operation of the University of Kentucky 1-Mev Van de Graaff generator. HOMER H. GIVIN, T. M. HAHN, JR., GEORGE C. PATTERSON, AND L. W. COCHRAN, *University of Kentucky.*

Focusing problems and voltage stabilization of the Kentucky Van de Graaff generator. J. L. RYAN, B. D. KERN, AND C. A. BAYSE, *University of Kentucky.*—The Kentucky 1-Mev Van de Graaff generator has been adapted so that exterior control of the electrostatic focusing is possible. Work is now in progress on the corona stabilization of the voltage. Other methods of voltage stabilization such as accelerating electrons up the differential pumping tube are being considered.

Quantiative measurements of beta-radiation. R. S. CASWELL, *University of Kentucky.*—An extrapolation ionization chamber with infinitely thick electrodes of the same composition but one electrode radioactive may be used to measure the rate of energy production by a beta-emitter. A 4π solid angle counter yields the absolute beta-disintegration rate. From these two measurements, the average beta-energy per disintegration may be obtained. All three quantities are of interest, especially in biological applications of radioactivity. Some results are discussed.

A fast coincidence analyzer. CHARLES C. RAYBURN, AND T. M. HAHN, JR., *University of Kentucky.*—A fast delayed analyzer was displayed and its operation described. This instrument was designed primarily for the half-life measurement of very short-lived isomers, although well suited to other experiments in which the measurement of short time intervals is desirable. Scintillations are produced in anthracene crystals, which are cemented with Canada balsam to RCA 5819 photomultiplier tubes. Pulses from the photomultipliers are delayed by RG 65/U coaxial cable. A shorted length of the same cable located at the coincidence circuit limits the pulse width. Pulse amplification was accomplished with 2.6×10^{-9}-sec rise time. Simultaneous excitation of the two counters was produced by annihilation gamma-quanta from Na^{22} positrons. Counting rates as a function of delay, using the mentioned source, provided the "prompt" resolution curve. This curve indicated the minimum half-life directly measurable to be 1.6×10^{-9} sec.

Following a luncheon in the college cafeteria, a business meeting was held at which the following officers were elected for 1951: *President*, EARLAND RITCHIE, *Centre College; Vice-President*, P. C. OVERSTREET, *Morehead State Teachers College; Representative to Executive Committee, AAPT*, RALPH A. LORING, *University of Louisville; Secretary-Treasurer*, RICHARD HANAU, *University of Kentucky.*

RICHARD HANAU, *Secretary*

Michigan Teachers of College Physics

On Saturday, April 21, 1951, teachers of physics from Michigan's colleges and universities met at Wayne University to inspect the quarters of the Physics Department in Science Hall and to hear a program of contributed papers. DR. EVERETT R. PHELPS, acting chairman of the Physics Department, presided at the meeting. One hundred guests registered for the meeting. A program for the ladies included a lecture on the topic "Life in Japan" and a tour of the Detroit Art Museum. Luncheon was served at the Student Center, at which PRESIDENT DAVID O. HENRY extended a welcome to the visitors for Wayne University.

In the two sessions the following papers were presented.

Sound experiments suitable for television. EDWARD BASCOM, *Wayne University.*

The resultant and the meter stick. D. A. NAYMIK, *Michigan State Normal.*

Neutron refractive indices. C. KIKUCHI, *Michigan State College.*

An electronic diffraction analyzer. K. R. SYMON, *Wayne University.*

Radiation intensities near a demonstration x-ray tube. R. SCHLEGEL, *Michigan State College.*

The transient characteristics of the tuba. ERVIN HOLLAND-MORITZ, *Wayne University.*

A spark timer and an impulse counter used as an inertia balance. H. PETTERSON, *Albion College.*

Reduction of test grades to standard distribution. H. BOWLDON, *Wayne University.*

The teaching of physics in Japan. M. S. WATANABE, *Wayne University.*

Pressure energy and Bernoulli's principle. G. A. LINDSAY, *University of Michigan.*

A paper by W. W. Sleator, *University of Michigan*, announced in the preliminary program was not given owing to Professor Sleator's illness and inability to attend.

B. H. DICKINSON